Second-order relativistic initial conditions for N-body simulations

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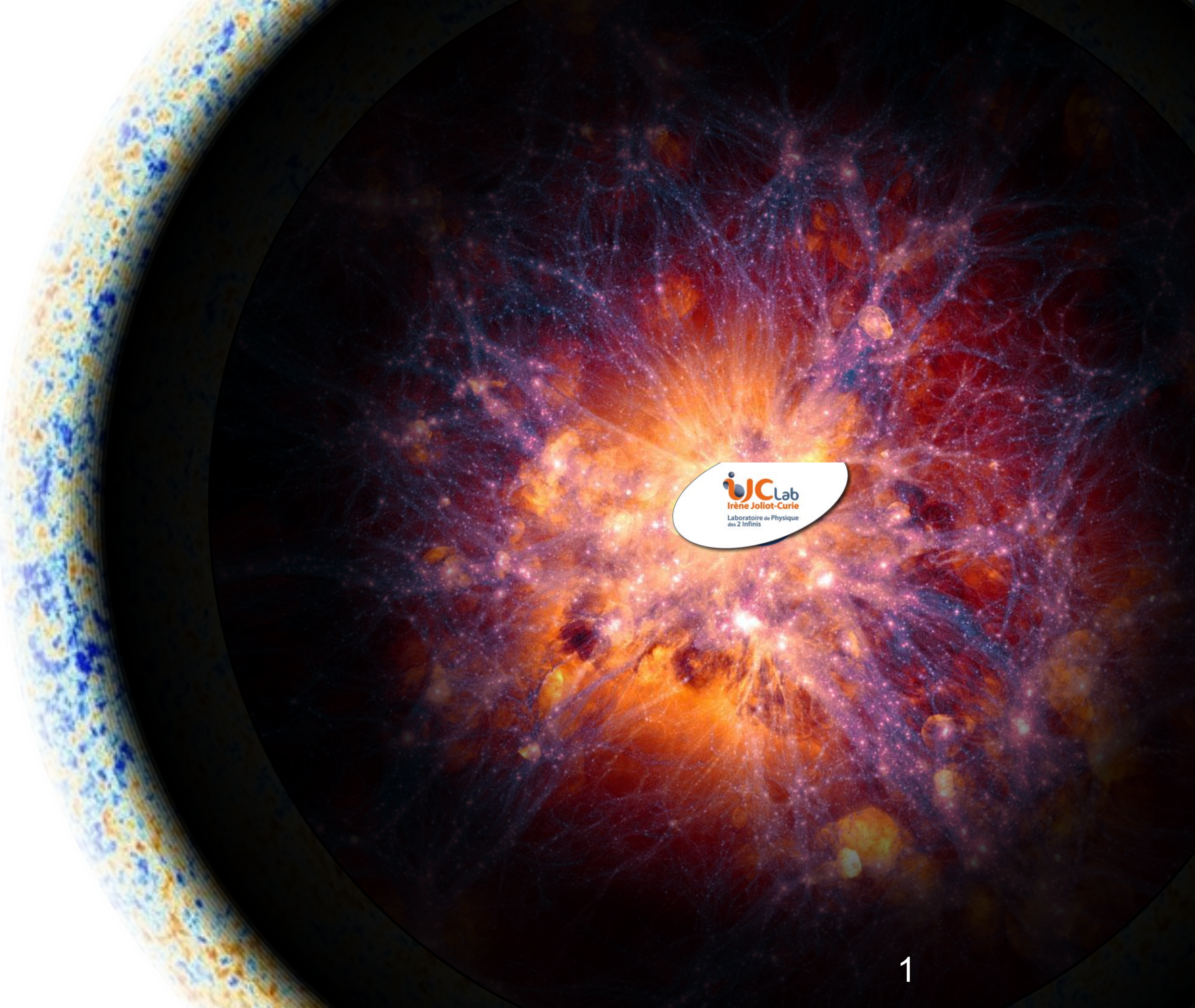
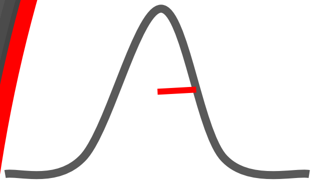


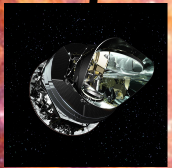
Inflation → Gaussian Perturbation



$\sigma_k \propto \sqrt{P(k)}$

$$P(k) = \frac{2\pi^2 A_s}{k^3}\left(\frac{k}{k_p}\right)^{n_s - 1}$$

$$C_\ell^{TT,EE,TE}$$



$$\ln 10^{10} A_s = 3.043 \pm 0.014$$

$$n_s = 0.9652 \pm 0.0042$$

Planck Collaboration
1807.06209

Inflation

Nearly-Gaussian Perturbation

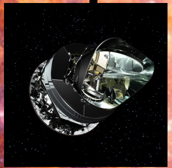
$B(k_1, k_2, k_3)$

$B^{IJK}_{\ell_1, \ell_2, \ell_3}$

Creminelli and Zaldarriaga
0407059

$$B(k_1, k_2, k_3 \to 0) = (n_s - 1)P(k_1)P(k_2)$$

$$f_{NL} \sim 10^{-2}$$



$$f_{NL} = -0.9 \pm 5.1$$

Planck Collaboration
1807.06209

Inflation

$P(k)$

$B(k_1, k_2, k_3)$

$f_{NL} = -12 \pm 21$

SDSS Collaboration
2106.13725

Future:

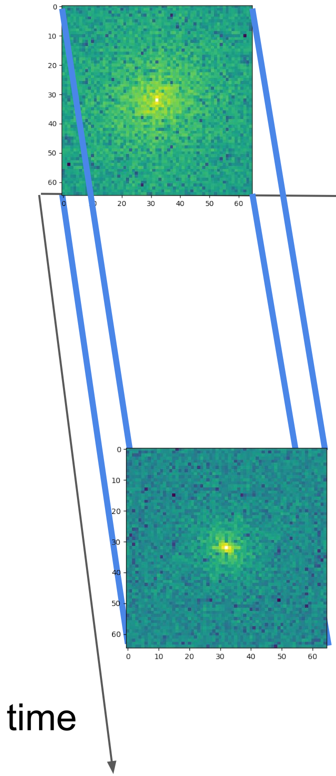
Euclid $\sigma_{f_{NL}} \sim 1$

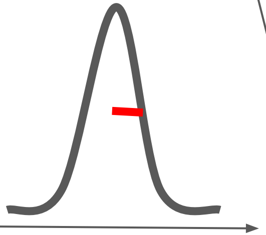
Desjacques et al 1611.09787

SPHEREx
SKA
$\sigma_{f_{NL}} \sim 0.1$

Karagiannis et al 1801.09280 and 1911.03964
Doré et al. 1412.4872

Linear evolution





Relativistic Poisson equation (G00=T00)

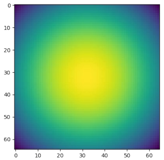
$$3\mathcal{H}\left(\phi' + \mathcal{H}\phi\right) + k^2\psi + \frac{3\mathcal{H}^2}{2}\delta = 0$$

Stochastic part simplifies !

Translation invariance = decoupled modes

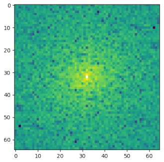
$$\mathcal{I}(t, \mathbf{k}) = T^{(1)}_{\mathcal{I}}(t, k)\mathcal{R}(\mathbf{k})$$

$$=$$

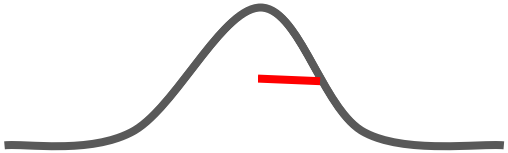


deterministic

$$\times$$



Stochastic



$$P(t, k) = \left(\mathcal{T}^{(1)}_{\mathcal{I}}(k)\right)^2 P(t = t_0, k)$$

Quadratic Evolution

First order equation with second order fields

Quadratic source term

$$3\mathcal{H}\left(\phi' + \mathcal{H}\phi\right) + k^2\psi + \frac{3\mathcal{H}^2}{2}\delta = 3\mathcal{H}^2\phi^2 - \frac{1}{2}\nabla_i\phi\nabla^i\phi + 2\phi\Delta\phi$$

$$\nabla_i\phi\nabla^i\phi \rightarrow \int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\left[-\mathbf{k}_1\cdot\mathbf{k}_2\right]\phi(\mathbf{k}_1)\phi(\mathbf{k}_2)\delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)$$

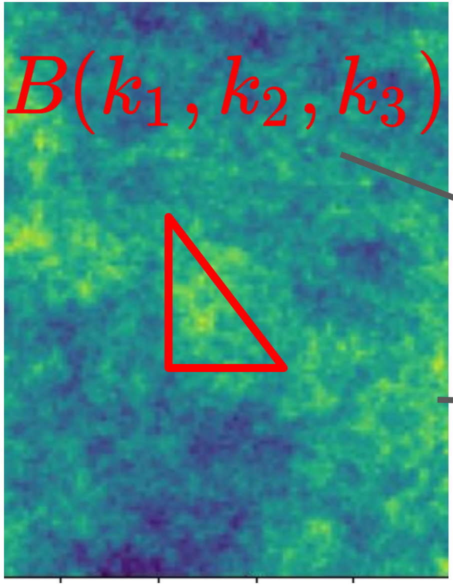
$$\phi\Delta\phi \rightarrow \int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\left[-k_2^2\right]\phi(\mathbf{k}_1)\phi(\mathbf{k}_2)\delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)$$

Quadratic level: the bispectrum evolution

$$\mathcal{I}(t,\mathbf{k}) = T^{(1)}(t,k)\mathcal{R}(\mathbf{k}) + \int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3} T^{(2)}(t,k_1,k_2,k)\mathcal{R}(\mathbf{k}_1)\mathcal{R}(\mathbf{k}_2) + \mathcal{O}(\mathcal{R}^3)$$

$$\langle \mathcal{R}(\mathbf{k}_1)\mathcal{R}(\mathbf{k}_2)\mathcal{R}(\mathbf{k}_3)\rangle = (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B(k_1,k_2,k_3)$$

$\mathcal{R}$



Linear evolution

Non-linear evolution

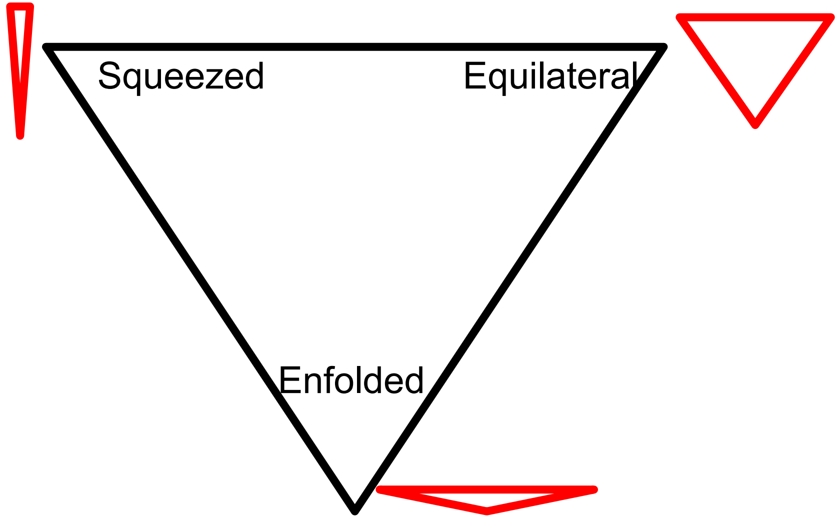
$$B(t,k_1,k_2,k_3) =$$

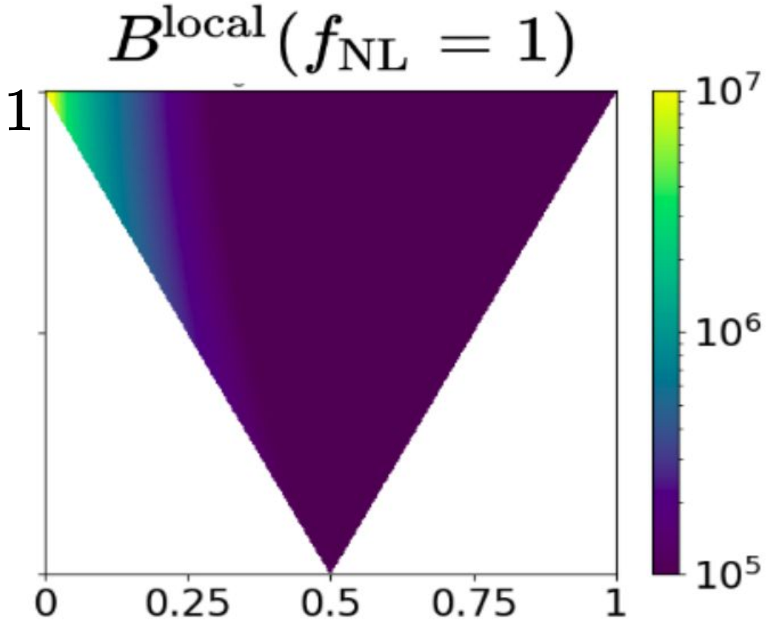
$$T^{(1)}(k_1)T^{(1)}(k_2)T^{(1)}(k_3)B_{\mathcal{R}}(k_1,k_2,k_3)$$

$$+2T^{(1)}(k_1)T^{(1)}(k_2)T^{(2)}(k_1,k_2,k)P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_2)$$

Squeezed limit

$$B(t,k_1,k_2,k_3) = T^{(1)}(k_1)T^{(1)}(k_2)T^{(1)}(k_3)B_{\mathcal{R}}(k_1,k_2,k_3)$$





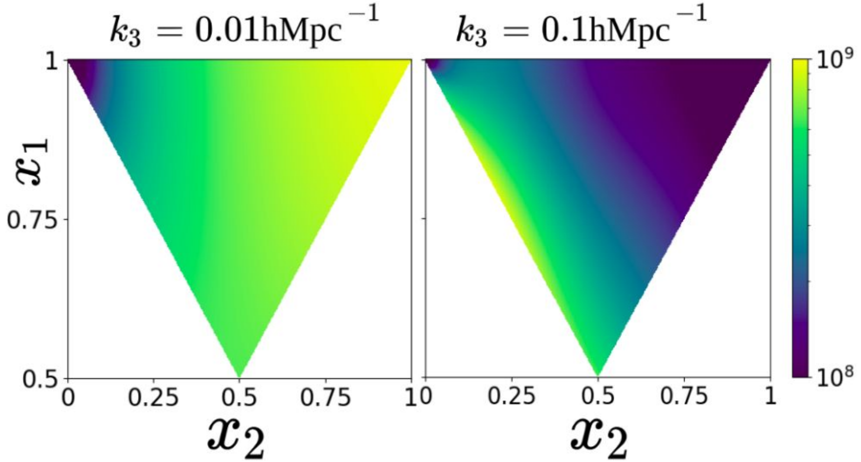
Single-field inflation = $f_{NL} \sim 10^{-2}$ ⇒ $f_{NL} \geq 0.1 =$ Multi-field inflation!

Relativistic effects

$$B(t,k_1,k_2,k_3) = \; +2T^{(1)}(k_1)T^{(1)}(k_2)T^{(2)}(k_1,k_2,k)P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_2)$$

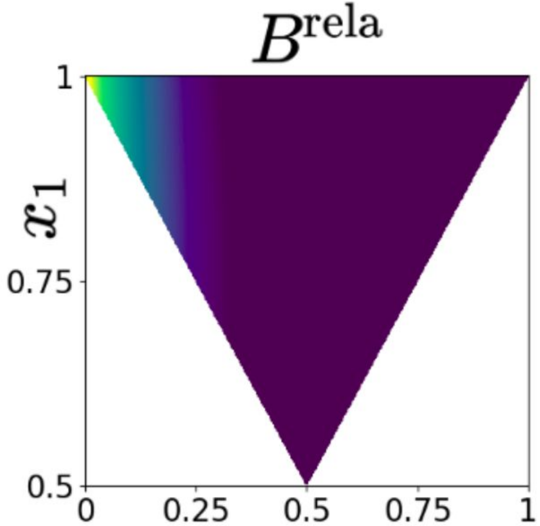
Newtonian limit:

- No horizon
- $\delta^{(1)} \propto a$
- $\delta^{(2)} \propto a^2$



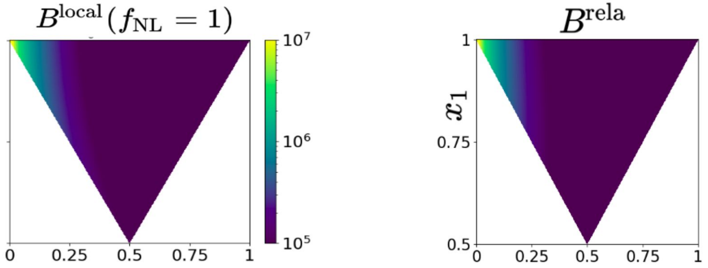
Relativistic effects: $\propto \mathcal{H}^2/k^2$

- Horizon $\mathcal{H}$
- $\delta^{(2)}_R \propto a^2\mathcal{H}^2/k^2 \propto a$

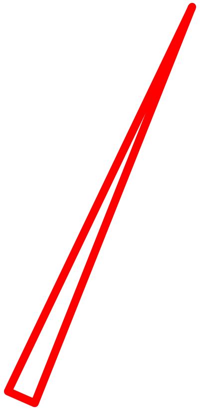


Relativistic effects

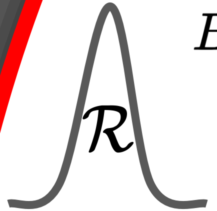
$$B(t, k_1, k_2, k_3) = T^{(1)}(k_1)T^{(1)}(k_2)T^{(1)}(k_3)B_{\mathcal{R}}(k_1, k_2, k_3) + 2T^{(1)}(k_1)T^{(1)}(k_2)T^{(2)}(k_1, k_2, k)P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_2)$$



- Degenerated in momentum space
- Degenerated in time
$$T^{(1)}(t) \propto a$$
$$T^{(2)}_R(t) \propto a$$
- Important for large scales, hence in the squeezed limit
- Small scales highly non-linear ⟶ N-body-simulation



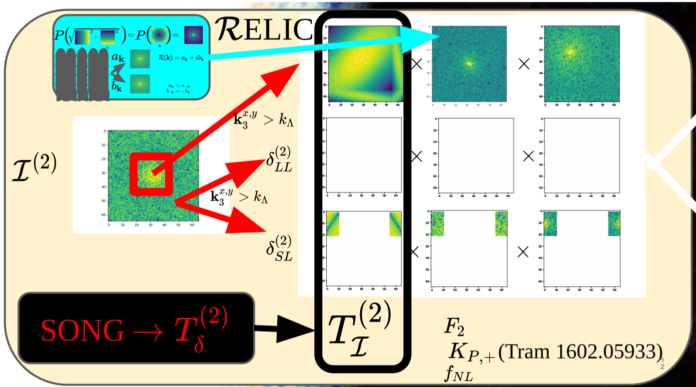
Inflation

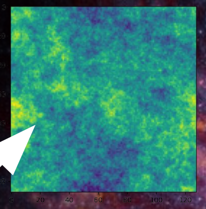


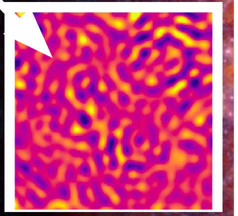
$$B(t, k_1, k_2, k_3) = 2T^{(1)}(k_1)T^{(1)}(k_2)T^{(2)}(k_1, k_2, k)P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k$$

Initial conditions for N-body simulation

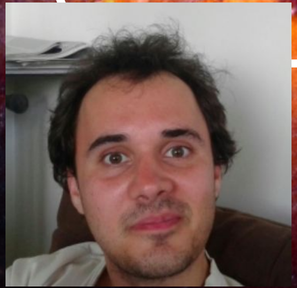
$$\mathcal{I}(t, \mathbf{k}) = T^{(1)}(t, k)\mathcal{R}(\mathbf{k}) + \int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}T^{(2)}(t, k_1, k_2, k)\mathcal{R}(\mathbf{k}_1)\mathcal{R}(\mathbf{k}_2)$$



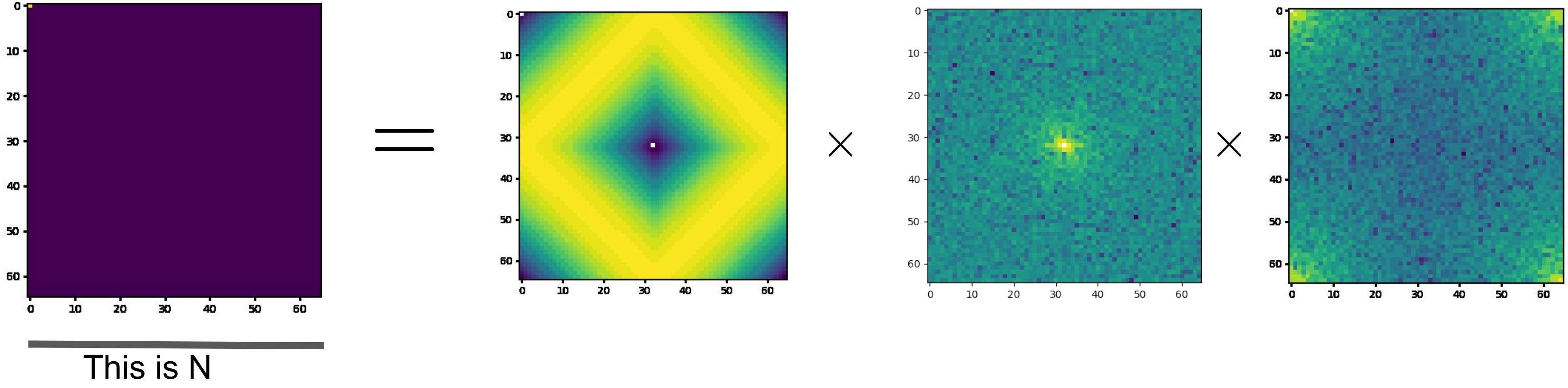




N-body sin
region

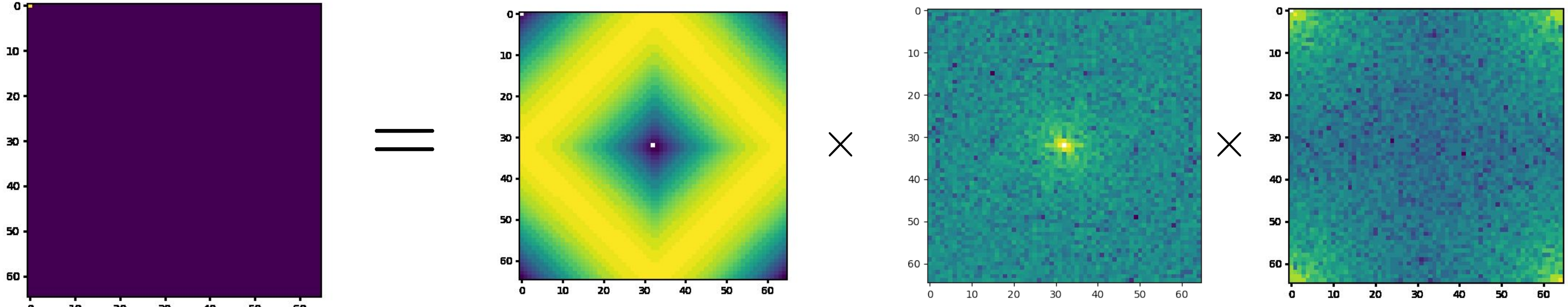


$$\mathcal{I}^{(2)}(\mathbf{k}_3) = \int \frac{d^3 k_1}{(2\pi)^3} T_{\mathcal{I}}^{(2)}(k_1, |\mathbf{k}_3 - \mathbf{k}_1|, k_3)\mathcal{R}^{(1)}(\mathbf{k}_1)\mathcal{R}^{(1)}(\mathbf{k}_3 - \mathbf{k}_1)$$

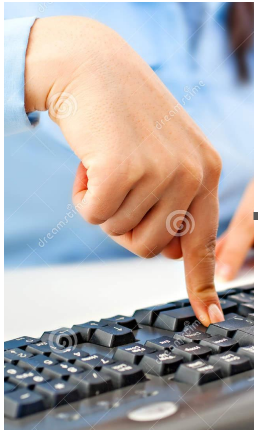


$$1 \text{ pixel} = N^3 \text{ terms} \longrightarrow N^3 \text{ pixels} = N^6 \text{ terms}$$

$$\mathcal{I}^{(2)}(\mathbf{k}_3) = \int \frac{d^3 k_1}{(2\pi)^3} T_{\mathcal{I}}^{(2)}(k_1, |\mathbf{k}_3 - \mathbf{k}_1|, k_3) \mathcal{R}^{(1)}(\mathbf{k}_1) \mathcal{R}^{(1)}(\mathbf{k}_3 - \mathbf{k}_1)$$



This is N



$\mathcal{O}(10^3)$ years

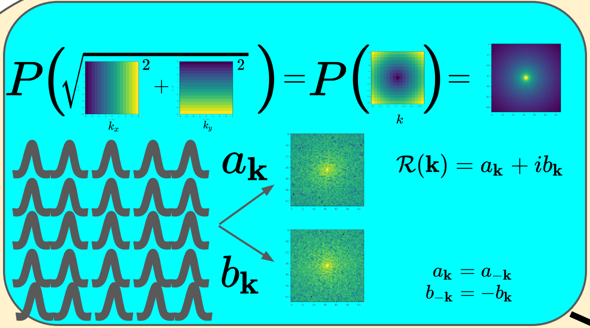
BAD

N^3 pixels $= N^6$ terms

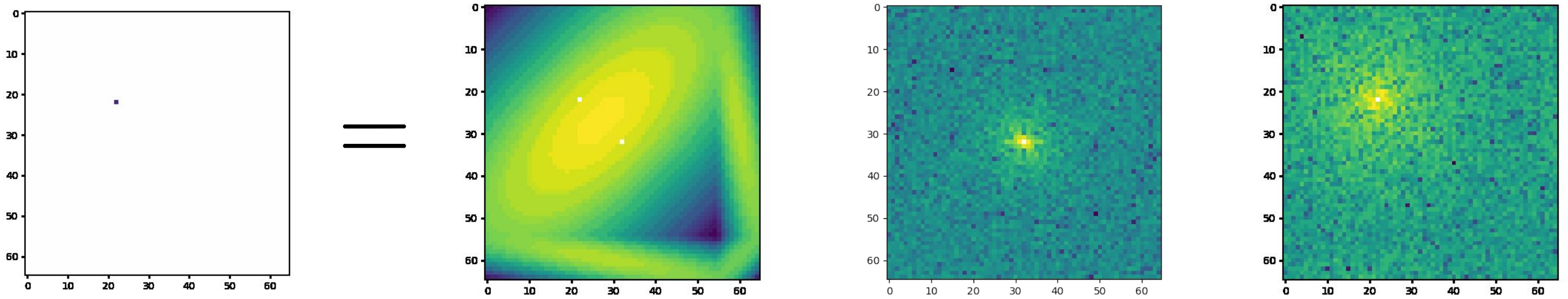
BAD BAD

$\mathcal{R}$ELIC

https://github.com/TomaMTD/gevolution-1.2

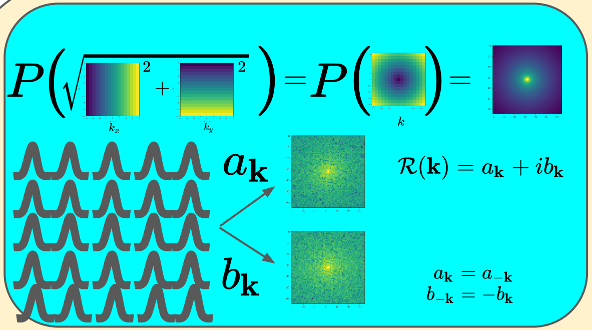


- Relativistic effects are important at large scale
- We introduce a cut-off k_Λ
- Exact computation for $\mathbf{k}_3^{x,y} < k_\Lambda$



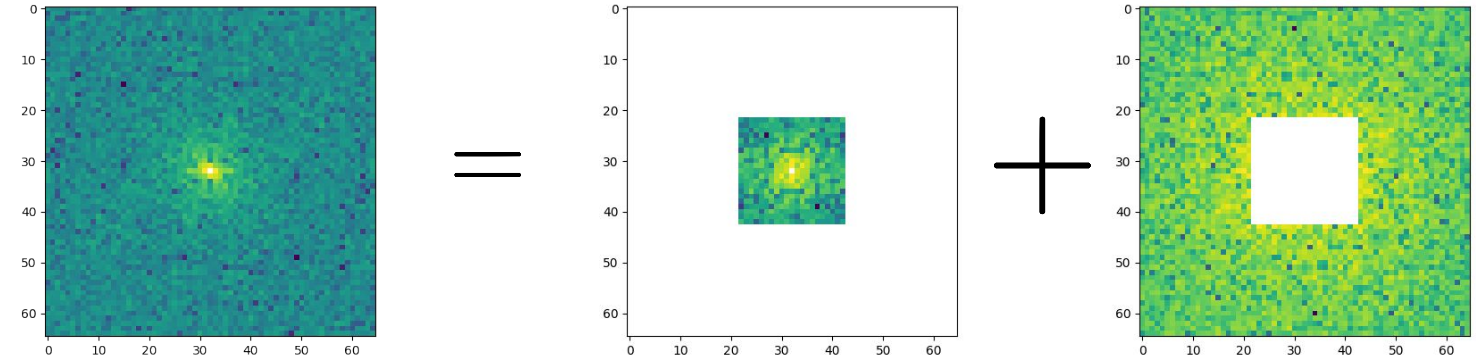
$$N_\Lambda^3 \text{pixels} \times N^3 \text{terms}$$

$\mathcal{R}$ELIC

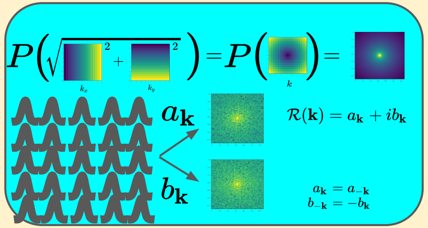


$\mathbf{k}_3^{x,y} > k_\Lambda$ Large/small scales splitting

$$\mathcal{R}^{(1)}(k) = W(k)\mathcal{R}^{(1)}(k) + (1 - W(k))R^{(1)}(k)$$

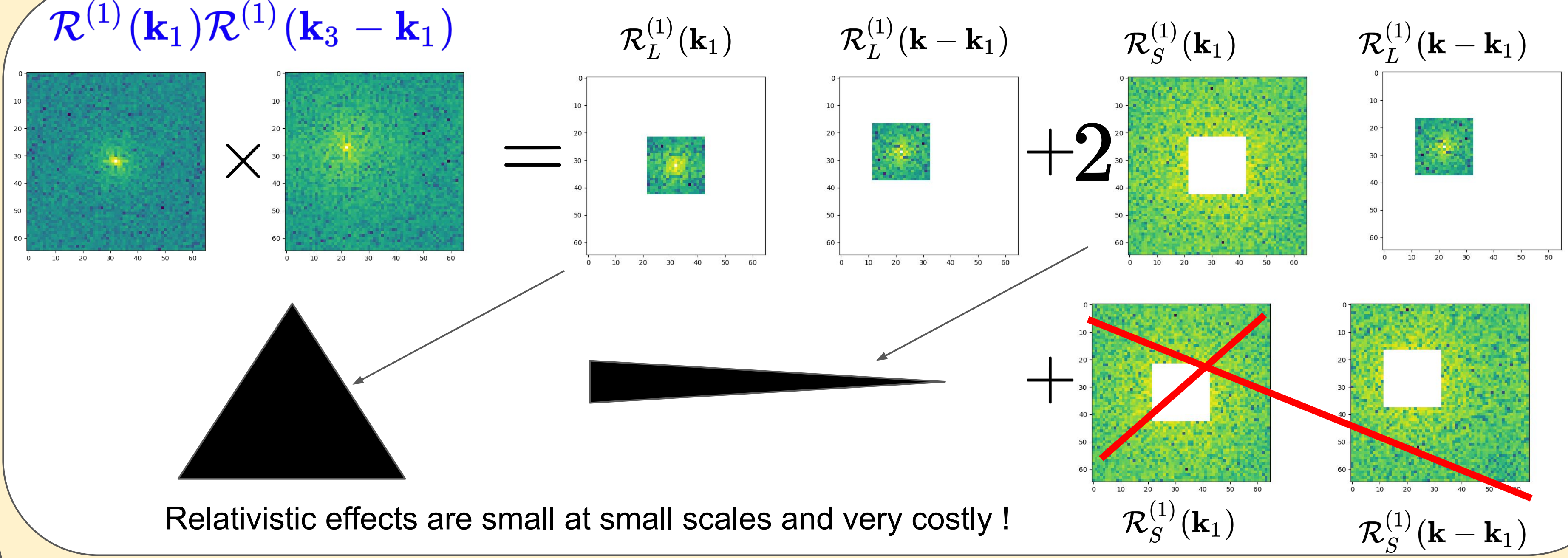


$\mathcal{R}$ELIC



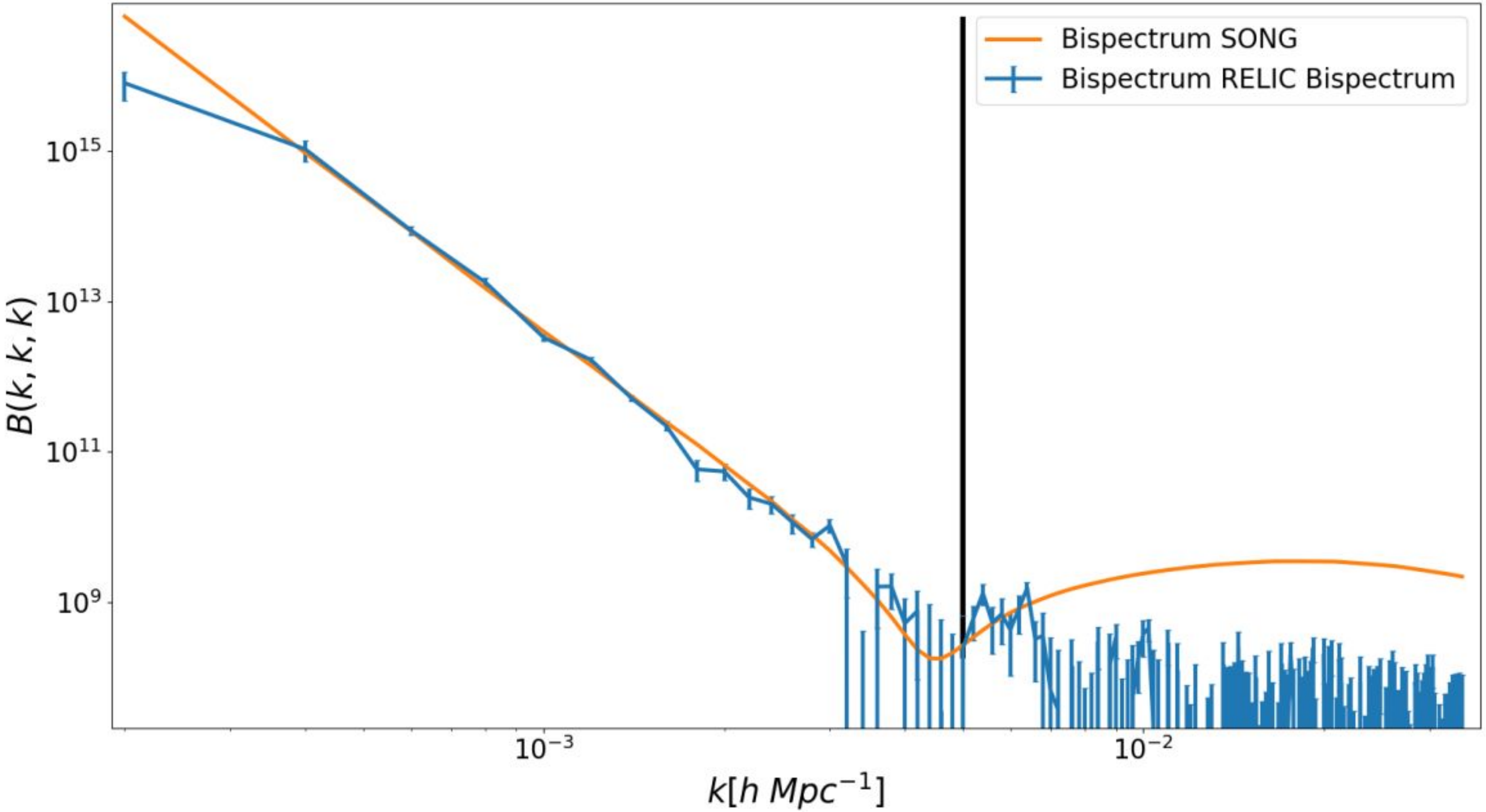
$\mathbf{k}_3^{x,y} > k_\Lambda$ Large/small scales splitting

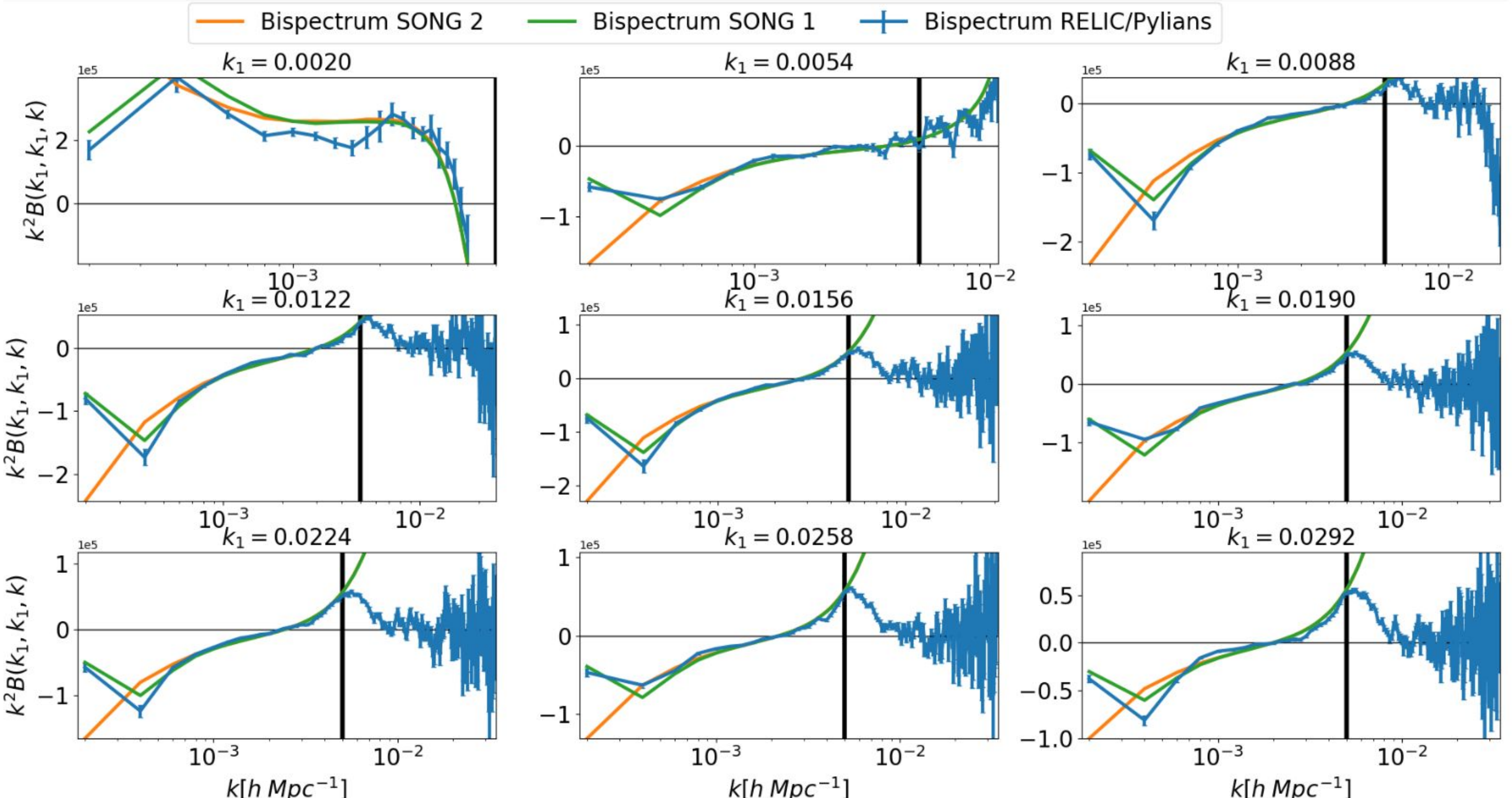
$$\mathcal{R}^{(1)} = \mathcal{R}_L^{(1)} + \mathcal{R}_S^{(1)}$$



Relativistic effects are small at small scales and very costly !

$$N^3 \text{pixels} \times N_\Lambda^3 \text{terms}$$


Bispectrum SONG
Bispectrum RELIC Bispectrum
10^{15}
10^{13}
10^{11}
10^{9}
$B(k, k, k)$
10^{-3}
10^{-2}
$k[h\ Mpc^{-1}]$



Bispectrum SONG 2
Bispectrum SONG 1
Bispectrum RELIC/Pylians
$k_1 = 0.0020$
$k_1 = 0.0054$
$k_1 = 0.0088$
$k_1 = 0.0122$
$k_1 = 0.0156$
$k_1 = 0.0190$
$k_1 = 0.0224$
$k_1 = 0.0258$
$k_1 = 0.0292$
$k^2B(k_1, k_1, k)$
$k[h\ Mpc^{-1}]$

Merci !