Theory and modelling of Compton light sources

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Instrumentation developments for High energy physics
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Outline

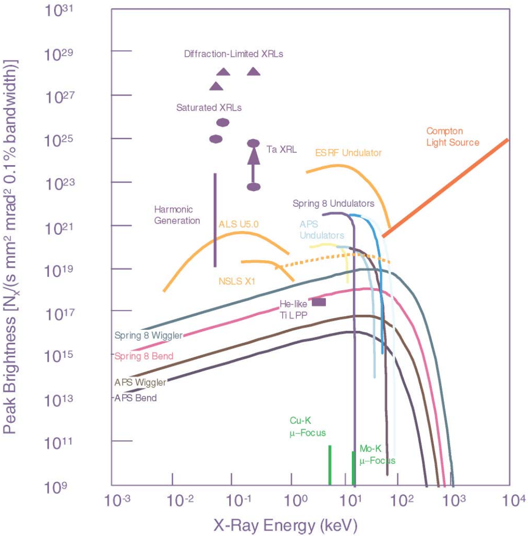
- Introduction
- Total yield and spectrum
- Smearing of real collimated spectrum
- Simulations on ThomX
- Summary and outlook

Inverse Compton Radiation

Compton effect (Nobel Prize of 1927 to Arthur Holly Compton): A gamma-ray photon loses a fraction of its energy when scattered off from an electron – reveals the quantum nature of photons.
The inverse Compton radiation: low energy laser photons increase their energy when scattered off from ultrarelativistic electrons.

Figure of Merit: Spectral Brightness

photon / s mm^2 mrad^2 (0.1% bandwidth)



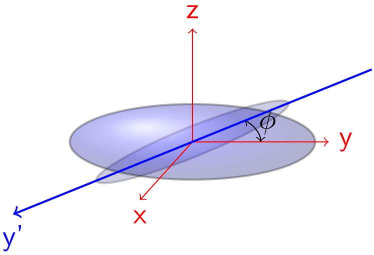
Energies needed for applications

- X-ray range from 10 keV to 120 keV (x-ray imaging)
- 'Soft gamma-rays' 1 MeV to 6 MeV (nuclear resonance fluorescence spectroscopy)
- Hard gamma-rays more than 10 MeV (polarized positrons production)

← F.V. Hartemann *et al* (2005)

Interaction of Electron Bunch with Laser Pulse

E. Bulyak and V. Skomorokhov, Phys. Rev. ST-AB 2005



yield / crossing

$$Y = \frac{N_{\rm las} N_e \sigma_{\rm C}}{2\pi\sqrt{\sigma'^2_z + \sigma^2_z}} \times \frac{1}{\sqrt{\sigma^2_x + \sigma'^2_x + \left(\sigma^2_y + \sigma'^2_y\right)\tan^2\phi/2}}$$

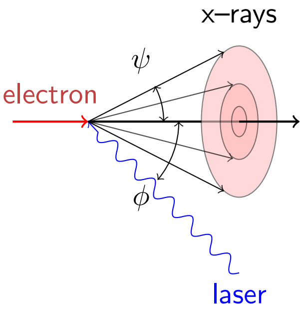
$\sigma'_{x,y,z}$, $\sigma_{x,y,z}$ are rms dimensions of the laser pulse and the electron bunch

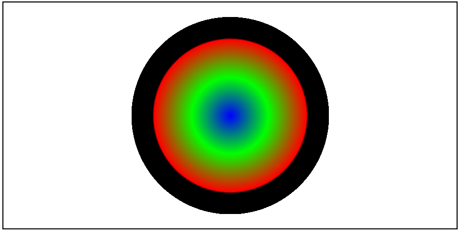
Yield is proportional to electron current !

Smaller ϕ – better !

Compton Radiation

Laser pulse = periodic structure (similar to undulator)





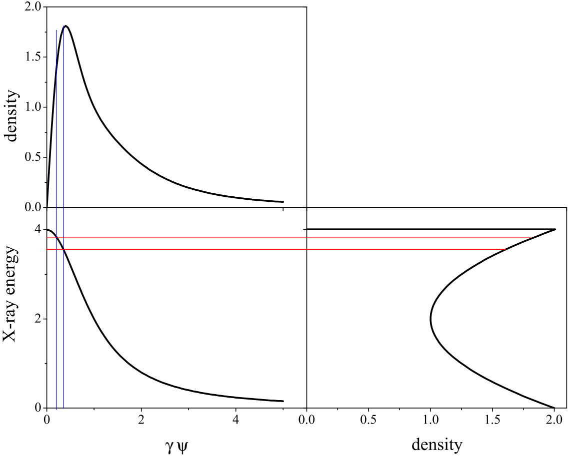
X–ray photon with a definite energy scattered off at the definite angle from electron's trajectory:

$$E_x \approx \frac{2\gamma^2(1+\cos\phi)E_{\rm las}}{1+\gamma^2\psi^2} = \frac{E_x^{\rm max}}{1+\gamma^2\psi^2}$$

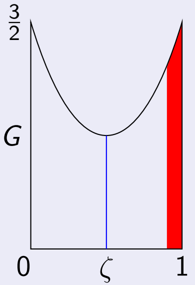
γ is the Lorentz-factor of the electron

Spectral–Angular Density Function

Ideal: zero emittances and spread



Spectrum Passed Collimator



Collimated ideal spectrum ⇒ "Quasi monochromatic"

$$G(\zeta) = 3[1/2 - \zeta(1-\zeta)]\times \left[\mathrm{H}\left(\zeta - \frac{1}{1+\chi_f^2}\right) - \mathrm{H}\left(\zeta - \frac{1}{1+\chi_i^2}\right)\right]$$

$$\chi_f \geq \chi_i\,; \qquad \chi \equiv \gamma\psi$$

the collimation angle scales as $1/\gamma$

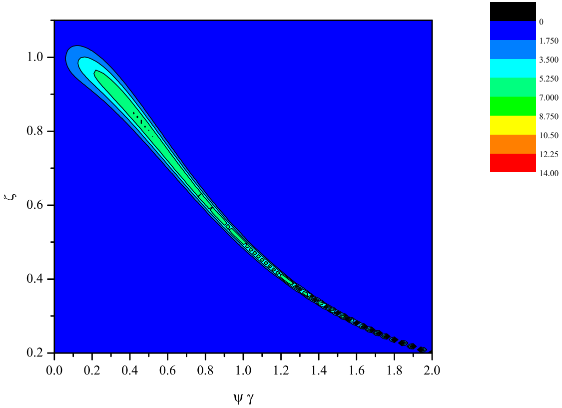
e.g. 0.1 % attained at $\psi \approx 31.6/\gamma$ mrad

Spectral brightness

$\approx 1.5 \times 10^{-3}\, Y$ phot/(0.1 % bandwidth)

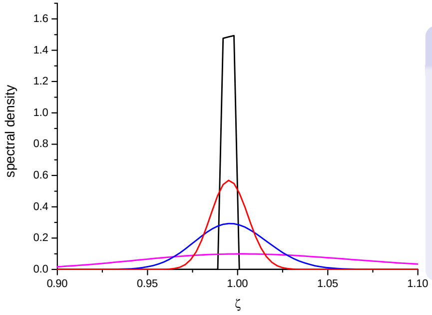
1. Smeared Spectral–Angular Density Function

Zero emittances, finite energy spread



Spectral angular density smeared off vertically

Spectrum Passed Collimator



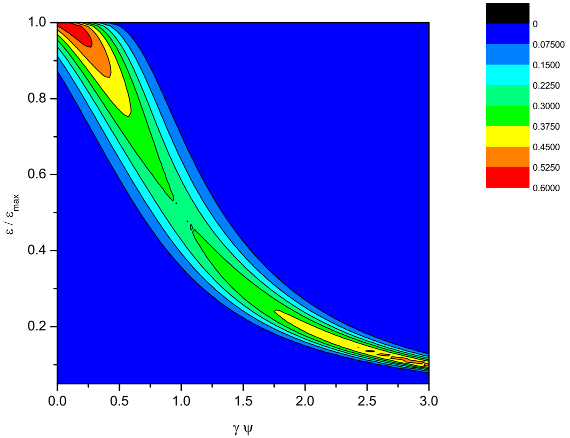
Collimated Spectrum

$$\mathcal{G}_\gamma(\zeta, x_\mathrm{i}, x_\mathrm{f}) = \frac{3}{4}\left[1 - 2\zeta\left(1-\zeta\right)\right] \times \left(\mathrm{Erf}\left(\eta_\mathrm{i}\right) - \mathrm{Erf}\left(\eta_\mathrm{f}\right)\right) \ ,$$

where $\eta_{\mathrm{i,f}} = -\left(1/\zeta - 1 - x_{\mathrm{i,f}}^2\right)/2\sqrt{2}s_\gamma$, $\mathrm{Erf}(z)$ the error integral function.

For pin-hole collimation energy spread of radiation is the doubled spread of electrons

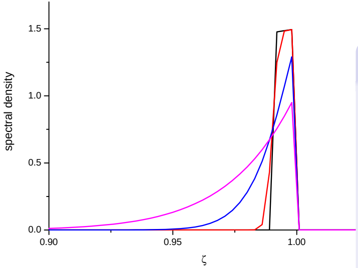
2. Smeared Spectral–Angular Density Function
Zero spread, finite emittances



Spectral angular density smeared off horizontally

Spectrum Passed Collimator

1D case (flat beam)



X-ray energy spectra for angular spread 0.01, 0.1, 0.5, 1.0 into collimating range 0...0.1.

2D beam (round) produces wider spectrum

Collimated Spectrum

$$\mathcal{G}_\psi(\zeta, x_\mathrm{i}, x_\mathrm{f}) = \frac{3\left[\zeta^2 + \left(1-\zeta^2\right)^2\right]}{2\sqrt{2\pi}s_\psi} \times$$

$$\left(\mathrm{erf}\left(\eta_\mathrm{f}^-\right) + \mathrm{erf}\left(\eta_\mathrm{f}^+\right) - \mathrm{erf}\left(\eta_\mathrm{i}^-\right) - \mathrm{erf}\left(\eta_\mathrm{i}^+\right)\right) \quad ,$$

with $\eta_{\mathrm{i,f}}^\pm = \left(x_{\mathrm{i,f}} \pm \sqrt{1/\zeta - 1}\right)/\sqrt{2}s_\psi$.

Small number of scattering per synchrotron period

E. Bulyak et al IPAC 2010

Energy rms spread (squared)

$$\sigma_p^2 = \frac{7}{10}\gamma\gamma_{\rm las}\ ,$$

where $\sigma_p \equiv \sqrt{\langle(\gamma_i-\gamma)^2\rangle}\Big/\gamma$;
$\gamma_{\rm las} = E_{\rm las}\Big/m_0c^2$.

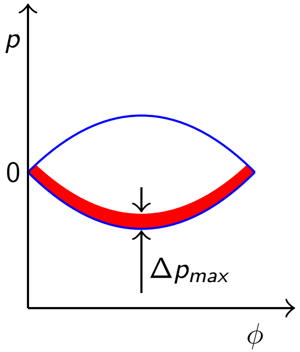
Transverse emittances:

$$\epsilon_{x,z} = \frac{3}{10}\beta_{x,z}^{(CP)}\frac{\gamma_{\rm las}}{\gamma}\ ,$$

with $\beta_{x,z}^{(CP)}$ magnitudes of the betatron functions at CP.
The transverse rms dimensions of the bunch at CP

$$\sigma_{x,z}^2 = \frac{\beta_{x,z}^2\gamma_{\rm las}}{3\gamma}\ .$$

Quantum lifetime



At the case of a small average energy of scattered–off photons as compared with the energy acceptance of the ring:

$$\gamma^2 \gamma_{\rm las} \sqrt{\frac{2\pi\eta h}{\gamma\gamma_{\rm rf}}} \ll 1 \;,$$

(η is the linear momentum compaction factor; h the harmonic number; $\gamma_{\rm rf} = eV_{\rm rf} \big/ m_0 c^2$ the reduced rf voltage) quantum losses:

$$\tau_{\rm qf}^{-1} = \frac{\omega_0 n_{\rm x}}{2\pi} \sqrt{\frac{9\gamma_{\rm rf}}{2\pi\gamma\eta h}} \exp\left(-\frac{3\gamma_{\rm rf}}{2\pi\gamma_{\rm las}\gamma^2\eta h}\right) \;.$$

with $n_{\rm x}$ the average number of scattered–off photons by each electron per turn, ω_0 the frequency of bunches circulation along the ring orbit.

Impact of synchrotron damping

$$s_{\text{asym}}^2 = \frac{s_s^2 \epsilon_s + s_c^2 \epsilon_c}{\epsilon_s + \epsilon_c}$$

ϵ – the partial average energy loss per turn

If Compton + SR take place, the resultant spread depends on the laser power

Evolution

$$s^2(t) = s^2_{\mathrm{asym}} + (s_0^2 - s^2_{\mathrm{asym}})\exp\left(-2t/\tau_s\right)$$

with τ_s the longitudinal damping time $\tau \approx \gamma/(\epsilon_s + \epsilon_c)$ [turns]
For the synchrotron dominated ring $\tau_s \simeq \mathrm{const}$

ThomX simulations: code

A code: Monte–Carlo, Monte–Carlo, Monte–Carlo . . .

input

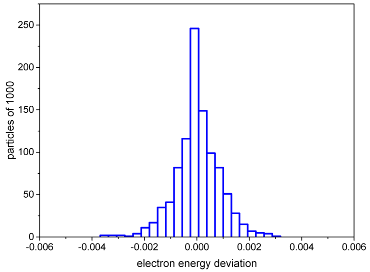
ALPHA 1 = 0.125D-1
BETA X = 0.1d0
BETA Z = 0.1d0
Q X = 3.17d0
Q Z = 1.74d0
emit0 x = 2.1d-6
emit0 z = 1.05d-6
EURF = 150.0d3
F RF = 500.0d6
HARM NUM = 30
E BEAM = 50.0d6
ELAS = 1.164d0
PHI X = 0.035
SYN LOSS = 3.0D0
SIGP X = 5.0D-5
SIGP Y = 3.0D-3
SIGP Z = 5.0D-5
P LAS = 0.02D0

output

MAX conversion 3.13E-004
MAX X energy (eV) 44565.0
Comp damping t 7164131.0
synch damping t 16666666.7
total damping t 5010414.87
Synch number 1.338E-002
ideal spread, 7/10 1.249E-002
emit Comp x 6.986E-010 m
emit Comp z 6.986E-010 m
averFI -1.187E-04
disp FI 3.96E-02
averP -2.104E-05
disp P 1.429E-03
x dim 1.66E-04
emit x 2.72E-07
z dim 1.146E-04
emit z 1.32E-07
total quanta scat 15.67

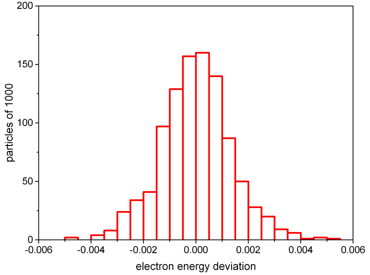
ThomX simulations: spectra

electron spectrum initial



at the beginning of 20 ms run

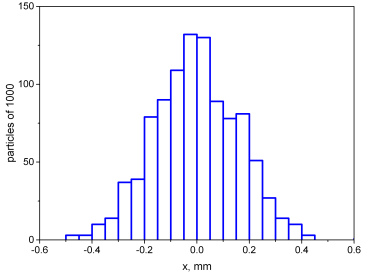
electron spectrum final



at the end of 20 ms run

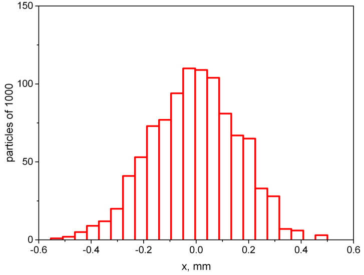
ThomX simulations: transverse

horizontal density initial



at the beginning of 20 ms run

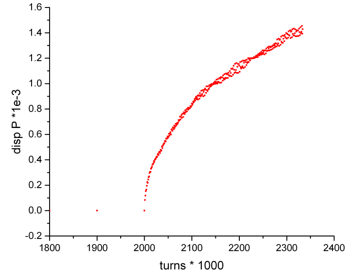
horizontal density final



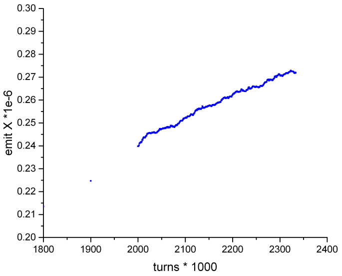
at the end of 20 ms run

ThomX simulations: evolution

energy dispersion in bunch

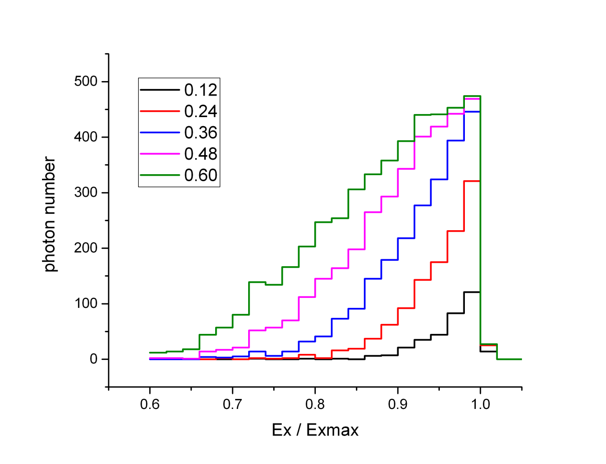


horizontal emittance



ThomX simulations: photon spectra

collimated x-ray spectra

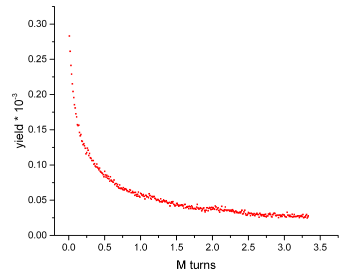


for the run at different $\gamma\psi$

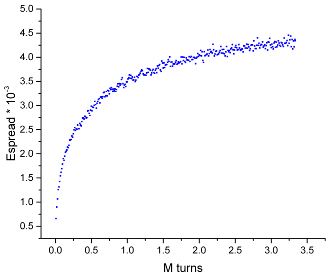
ThomX simulations: 200 ms (5 Hz)

initial δ phase density distribution

yield per turn



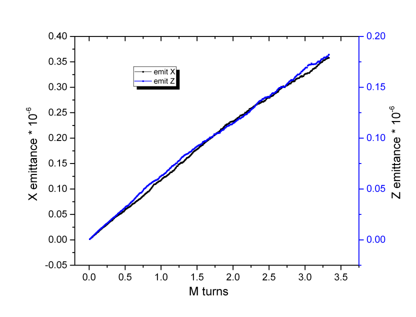
spread



ThomX simulations: 200 ms (5 Hz)

initial δ phase density distribution

emittances

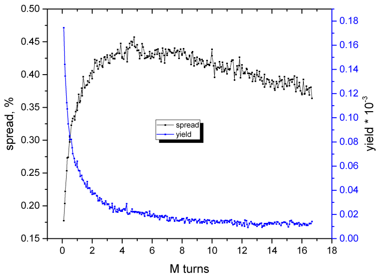


output

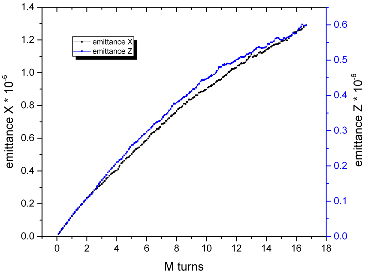
- total quanta scattered 188 / electron
- passed through collimator 57.23 / electron
- synchrotron dominated dynamics
- total photons scattered 5.87×10^{12}

ThomX simulations: 1 sec
initial δ phase density distribution

dispersion and yield

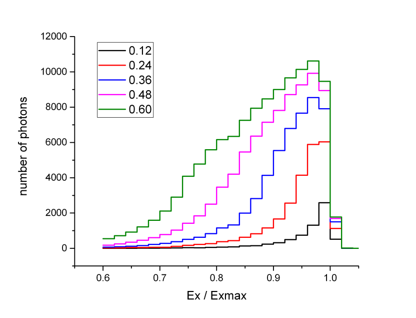


transverse emittances



ThomX simulations: 1 sec
initial δ phase density distribution

collimated spectra



output for 1 sec

- total quanta scattered 1026.75 / electron
- passed through collimator 394 / electron
- synchrotron dominated dynamics
- total photons scattered 6.4×10^{12}
- collimated $\gamma\psi = 0.6$ is 2.46×10^{12}

Summary and outlook

Run ThomX02j: total photons scattered $15.67 \times 50 \times 6.2415 \times 10^9 = 4.89 \times 10^{12}$ per second at 20 mJoul and 0.035 crossing angle
1.44×10^{12} passed through $\gamma\psi = 0.6$ collimator

Compton source ThomX will produce competitive X–ray beam