

Nuclear incompressibility from the viewpoint of spherical and deformed QRPA calculations



Gianluca Colò

Collaborators: D. Gambacurta, A. Pastore, V.O. Nesterenko, J. Kvasil, W. Kleinig

GMR Workshop

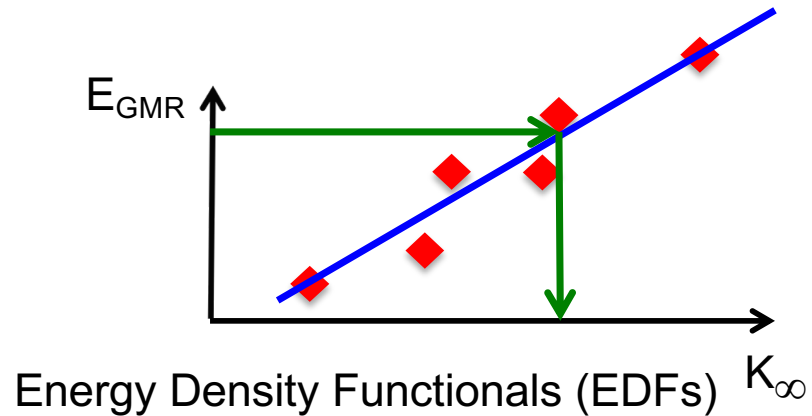
(GDR RESANET, 11/2020)



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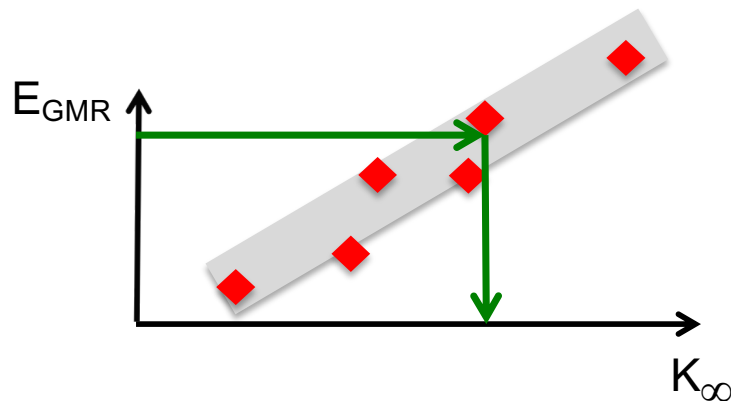


Background



Only self-consistent calculations that treat **uniform matter** and the **response of finite nuclei on equal footing** allow extracting K_{∞}

J.P. Blaizot, Phys. Rep. 64, 171 (1980)



There are different sources of model dependence in this procedure.

One **key point** is that different EDFs have different assumptions for the density dependence.

GC *et al.*, Phys. Rev. C70 (2004) 024307.



- Sensitivity to the choice of the nucleus?
- Can we understand what happens if we apply this procedure to superfluid/deformed nuclei?

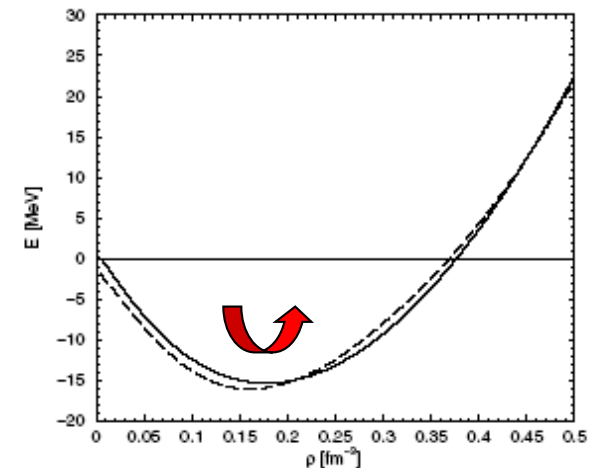
(Q)RPA using EDFs in a nutshell

$$E = \langle \Psi | \hat{H} | \Psi \rangle = \langle \Phi | \hat{H}_{eff} | \Phi \rangle = E[\hat{\rho}]$$

$|\Phi\rangle$ Slater determinant $\Leftrightarrow \hat{\rho}$ 1-body density matrix

$H_{eff} = T + V_{eff}$. If V_{eff} is well designed, the resulting g.s. (minimum) energy can fit experiment at best.
Hartree-Fock or Kohn-Sham.

- Within a time-dependent theory (TDHF), one can describe harmonic oscillations around the minimum.

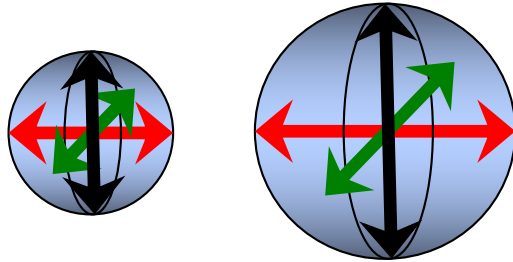


- The restoring force is: $v \equiv \frac{\delta^2 E}{\delta \rho^2} \cdot X_{ph} |ph^{-1}\rangle - Y_{ph} |hp^{-1}\rangle$
- The linearization of the equation of the motion leads to RPA¹.

¹Random Phase Approximation.

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \hbar\omega \begin{pmatrix} X \\ Y \end{pmatrix}$$

Incompressibility of nuclear matter from ^{208}Pb



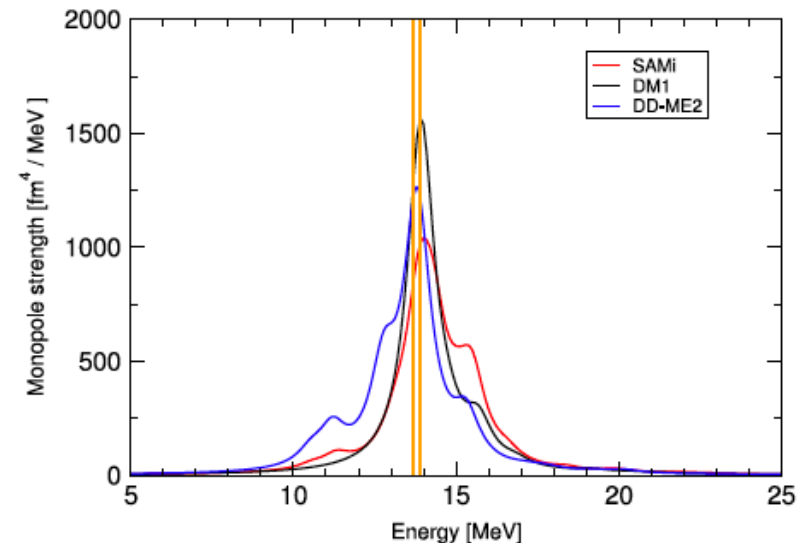
The compression-mode giant resonances and nuclear incompressibility

Umesh Garg^a, Gianluca Colò^{b,c,*}

Progress in Particle and Nuclear Physics 101 (2018) 55–95

240 ± 20 MeV from the GMR in ^{208}Pb

EDF	SAMi	DM1	DD-ME1
K_∞	245 MeV	225 MeV	245.5 MeV



Non-magic nuclei may (?) point to lower values of the nuclear incompressibility

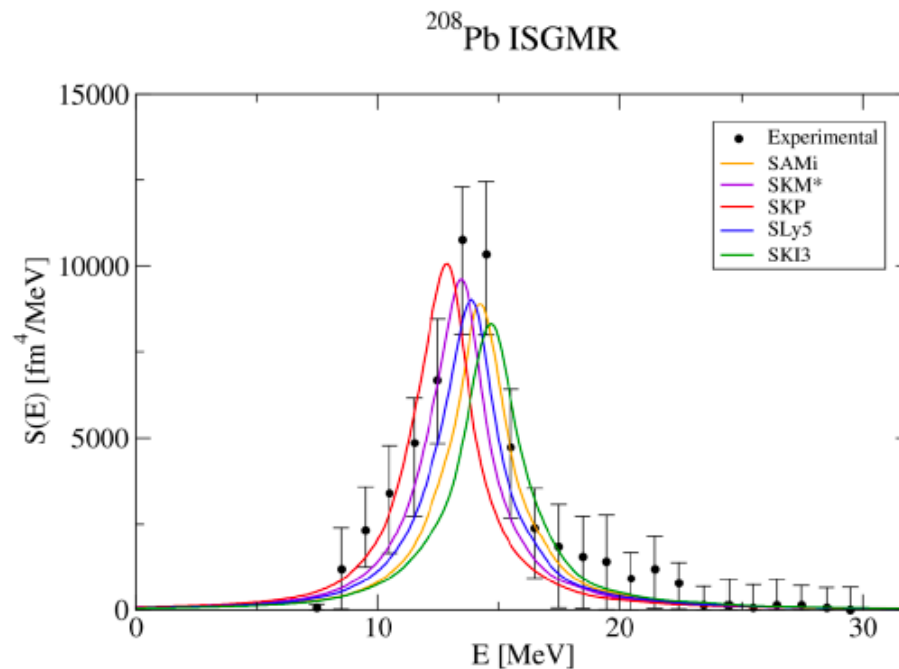


A different kind of analysis

by S. Zaghet



POLITECNICO
MILANO 1863



χ^2 made on:
 $E = 7.5 - 29.5$ MeV
(23 points)

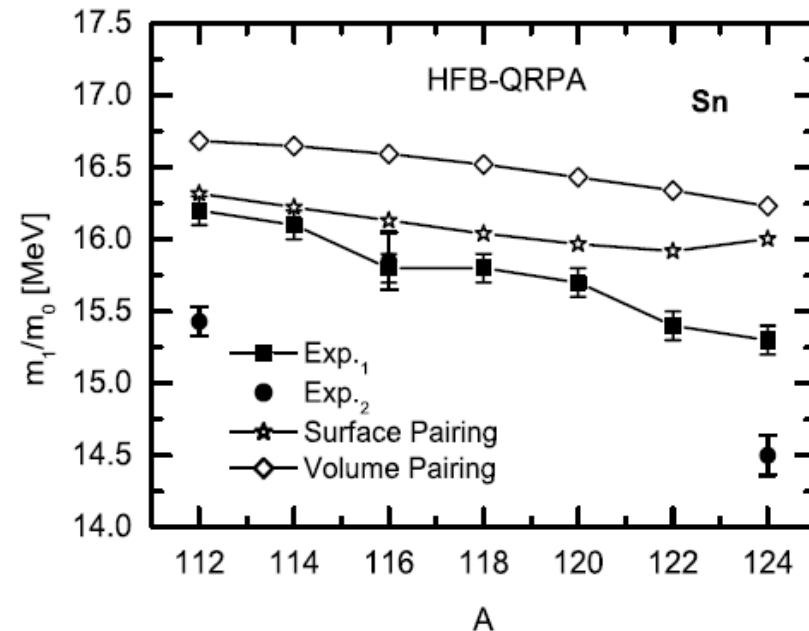
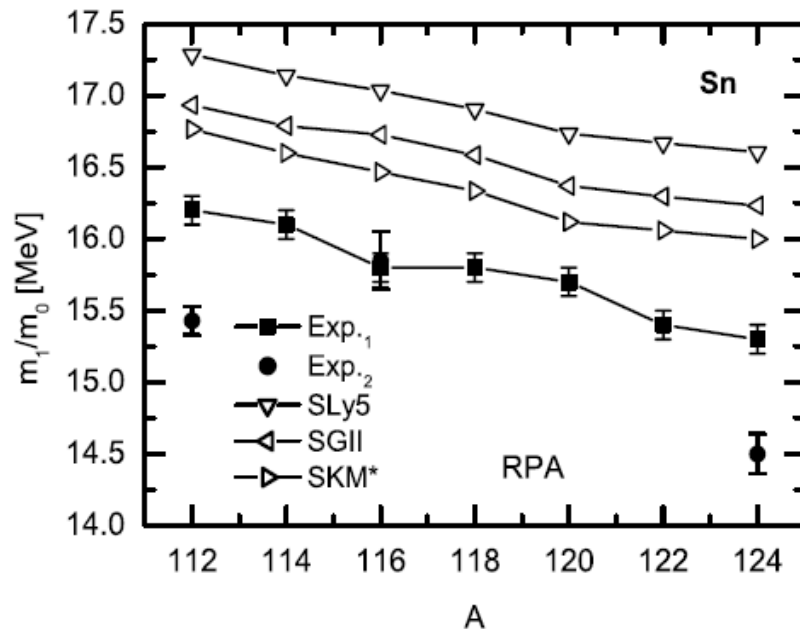
$K_\infty = 220 - 230$ MeV



	SKP	SKM*	SLy5	SAMI	SKI3
χ^2	68.6	35.7	31.5	44.6	73.8
reduced χ^2	3.0	1.6	1.4	1.9	3.2
K_∞ [MeV]	201	217	230	245	258



Superfluid nuclei



Pairing correlations lower the energy of the GMR (right panel) with respect to RPA (left panel).

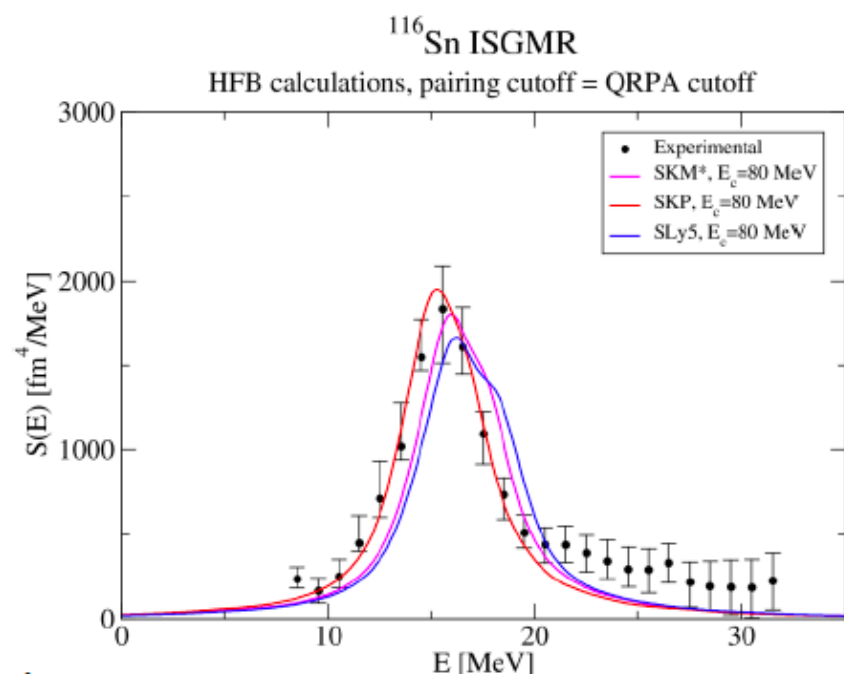
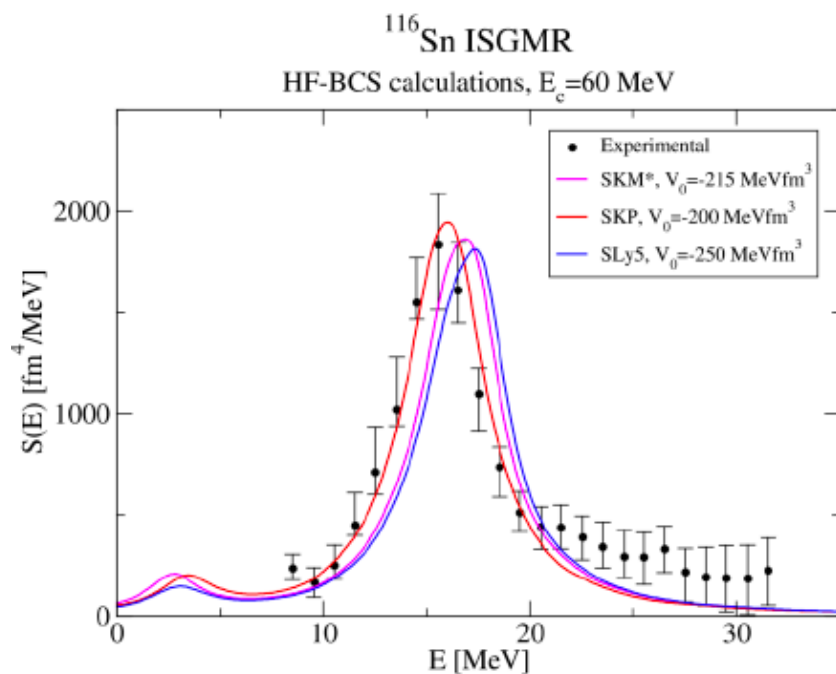
J. Li *et al.*, Phys. Rev. C 78, 064304(2008); L. Cao *et al.*, Phys. Rev. C 86, 054313 (2012).

The effect on the incompressibility can be of the order of 10%.

E. Khan *et al.*, Phys. Rev. C 82, 024322 (2010).



- Quantitative results depend on the kind of pairing force
- The information from g.s. pairing gap does not fully constrain it



χ^2 made on:
 $E = 8.5 - 19.5$ MeV
(12 points)

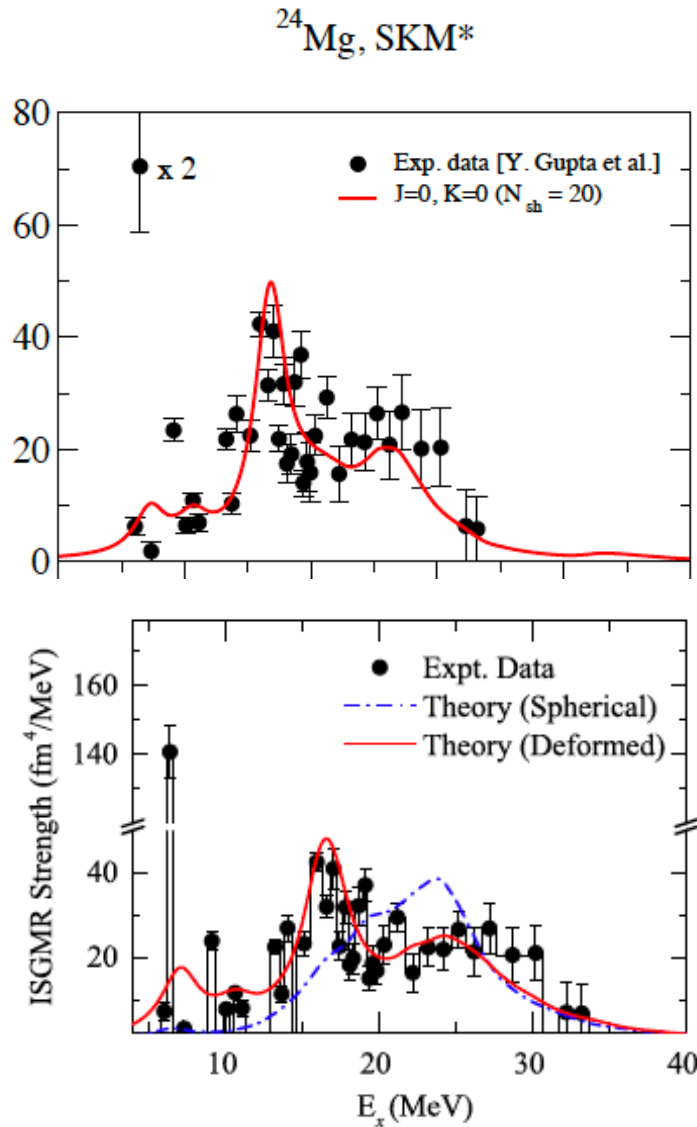
	SKP	SKM*	SLy5
$\chi^2_{\text{HF-BCS}}$	10.3	68.9	104.0
reduced $\chi^2_{\text{HF-BCS}}$	0.9	5.7	8.7
K_∞ [MeV]	201	217	230

$K_\infty \approx 200$ MeV

	SKP	SKM*	SLy5
χ^2_{HFB}	4.5	35.2	65.3
reduced χ^2_{HFB}	0.4	2.9	5.4
K_∞ [MeV]	201	217	230



Well-deformed nuclei



We compare with RCNP data from Y. Gupta *et al.*, PRC 93, 044324 (2016).

The two-peak structure is evident.

Thanks to K. Howard.

Similar calculations by K. Yoshida was used to show that the double peak is related to deformation.

General understanding: in deformed systems K^π are the only good quantum numbers. The $K=0$ components of the GMR and GQR are coupled and they both show up.

Deformed QRPA implementations

- Either **HFB or HF-BCS equations** with a Skyrme force and a pairing force are solved on a (HFBTHO / SKYAX).

M. Stoitsov *et al.*, Comp. Phys. Comm. 184 (2013) 1592;
P.-G. Reinhard, SKYAX (unpublished).

- This allows to determine study the potential energy surfaces (PESs).

$$E = E(\beta)$$

- The **QRPA equations** are solved on a discrete basis with **good K^π** .
- Using two different QRPA approaches strengthens our conclusions.

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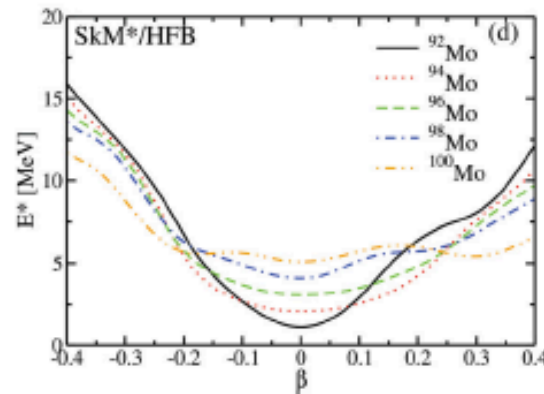
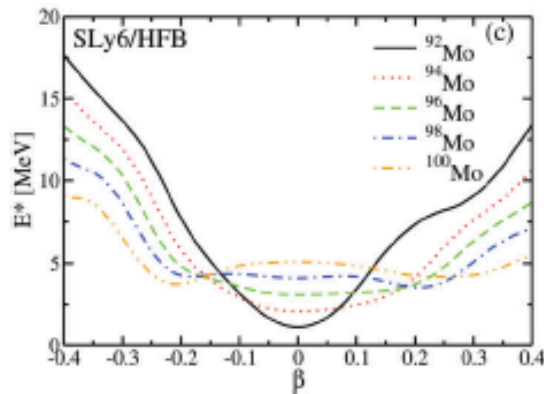
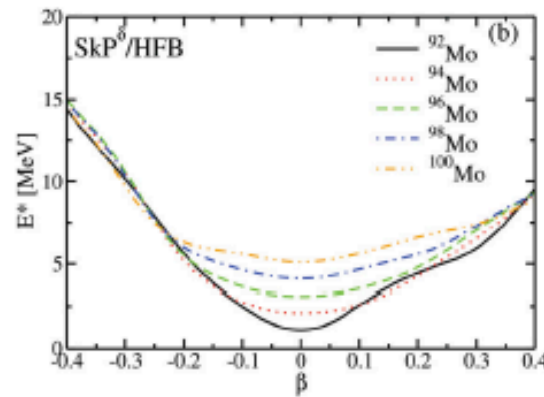
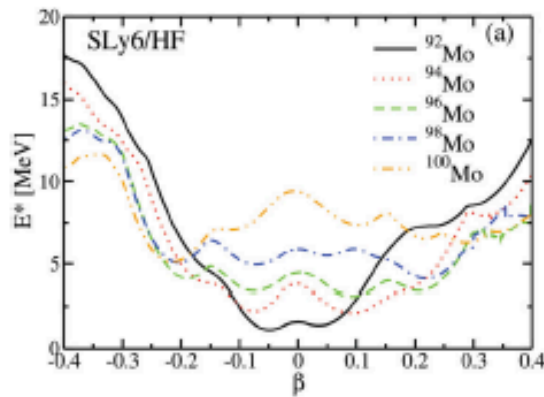


Isoscalar monopole and quadrupole modes in Mo isotopes:
Microscopic analysis

Gianluca Colò^{a,b,*}, Danilo Gambacurta^{c,d}, Wolfgang Kleinig^e, Jan Kvasil^f,
Valentin O. Nesterenko^{e,g,h}, Alessandro Pastoreⁱ



Potential energy surfaces of ^AMo



Pairing and deformation are **linked to each other**, we will not be able to disentangle the effect of either effect in the monopole strength.

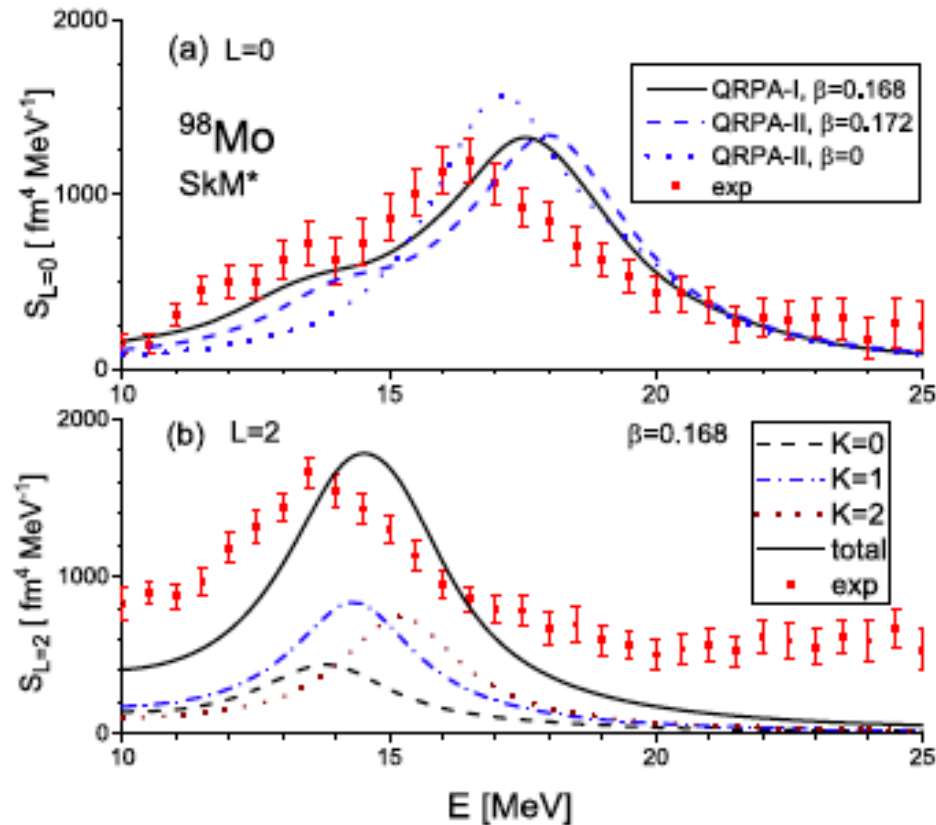
With increasing A, the curves become more flat (**“quadrupole softness”**).

It is hard to obtain a straightforward indication of the g.s. deformation from experiment.

$$\beta = 0.109, 0.151, 0.172, 0.168, 0.162 \quad (A = 92 \dots 100)$$



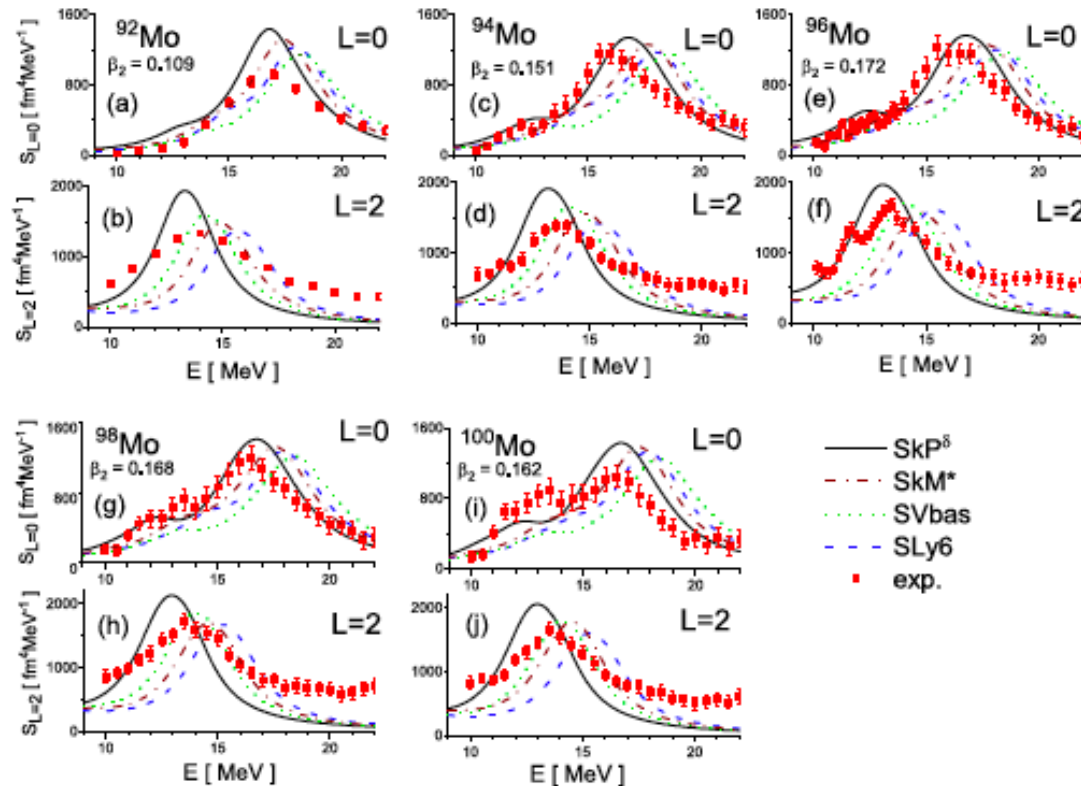
Effect of (modest) deformation



Implementing QRPA in two different manners we do not find results that change significantly.

Even a small deformation can shift the ISGMR by around 1 MeV.

Monopole and quadrupole strength in ^AMo



The “shoulder” is due to the monopole-quadrupole coupling.

The Skyrme EDF that better reproduces the GMR (GQR) results is SkP^δ (SVbas).

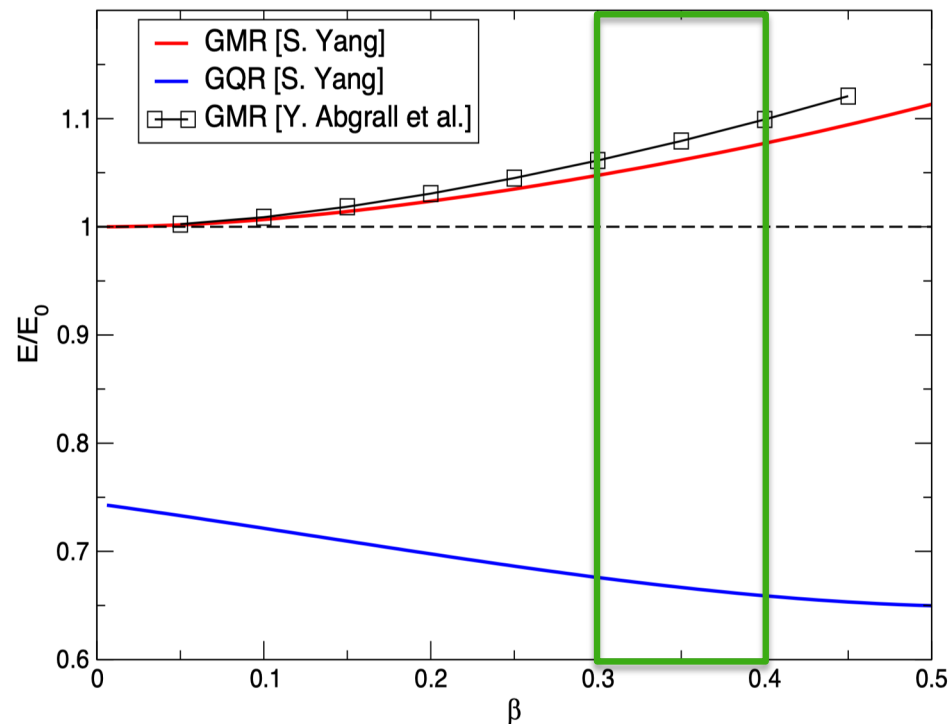
Warning, warning ...

Incompressibility K_∞ and isoscalar effective mass m^*/m for the Skyrme forces SVbas, SLy6, SkM*, and SkP^δ .

	SVbas	SLy6	SkM*	SkP^δ
K_∞ [MeV]	234	230	217	202
m^*/m	0.9	0.69	0.79	1



In macroscopic models, one starts from the “ideal” GMR and GQR of spherical nuclei and finds that two peaks arise from the **coupling of the K=0 components of GMR and GQR**.



- S. Yang, NPA 401 (1983) 303

Hydrodynamical calculations

$$E_0 (1 + 0.86\delta^2 - 1.25\delta^3)$$

- Y. Abgrall *et al.*, NPA 436 (1980) 431

Adiabatic cranking model

Quite similar results!

- T. Suzuki and D. Rowe, NPA 289 (1977) 461
- Y. Shimizu and K. Matsuyanagi, PTEP 72 (1984) 1017
- S. Åberg, NPA 473 (1987) 1
- D. Zawischa *et al.*, NPA 311 (1978) 445



Conclusions

- Personal (or shared?) belief: **the nuclear incompressibility should be around 240 MeV**, if we base ourselves on few magic nuclei.
- In some cases (e.g. ^{90}Zr), experimental uncertainties play the largest role.
- **Superfluid nuclei may point to a lower value** but our knowledge of pairing is not accurate enough to say more.
- **We may need to fit deformed nuclei into the picture.**
- **The coupling between the $K=0$ components of the monopole and quadrupole** plays a role in determining the peak of the monopole strength.
- Perspective: isotopic/isotonic chains in which open questions could be benchmarked.



Thank you!

