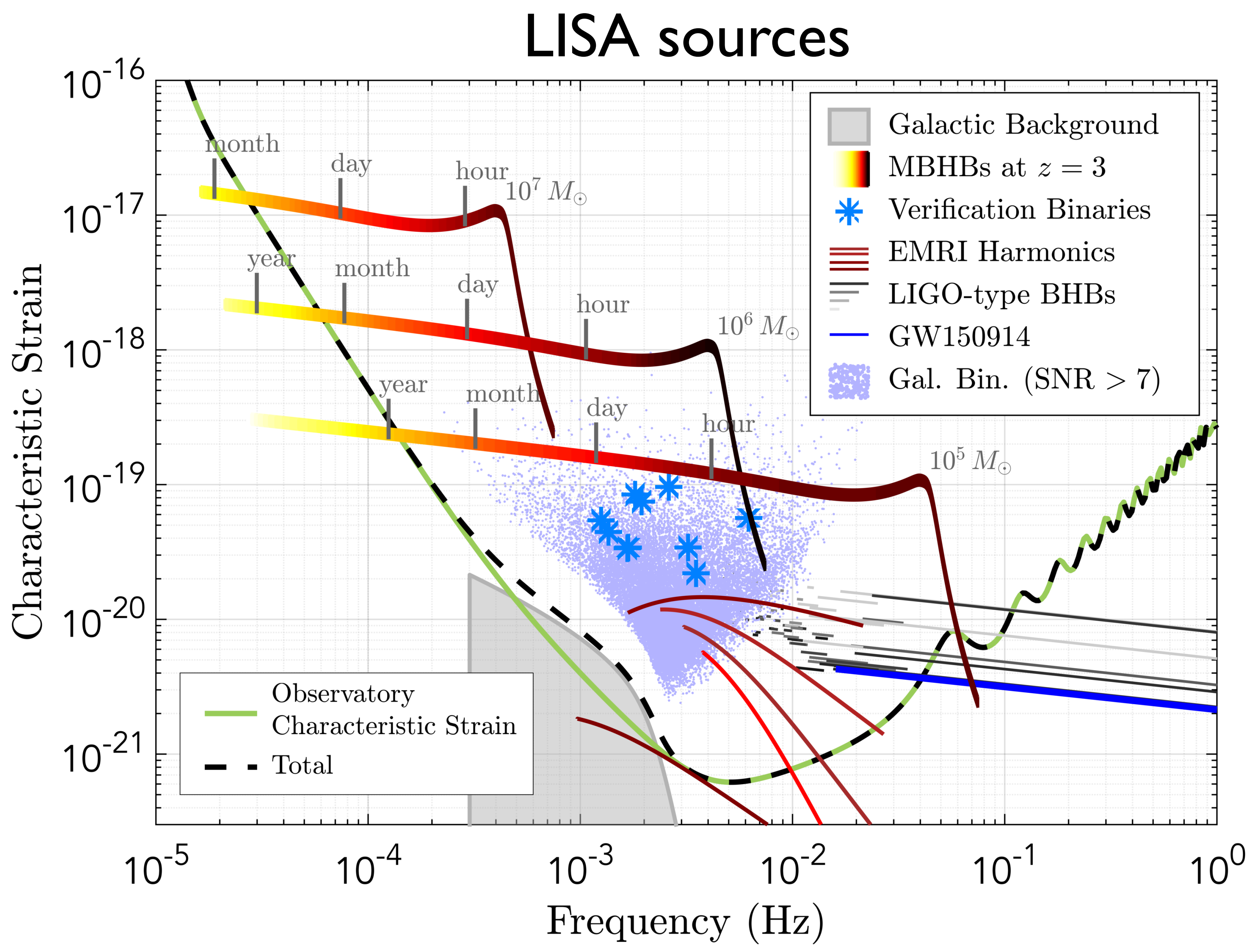
Simulating the parameter recovery of massive binary black holes with LISA

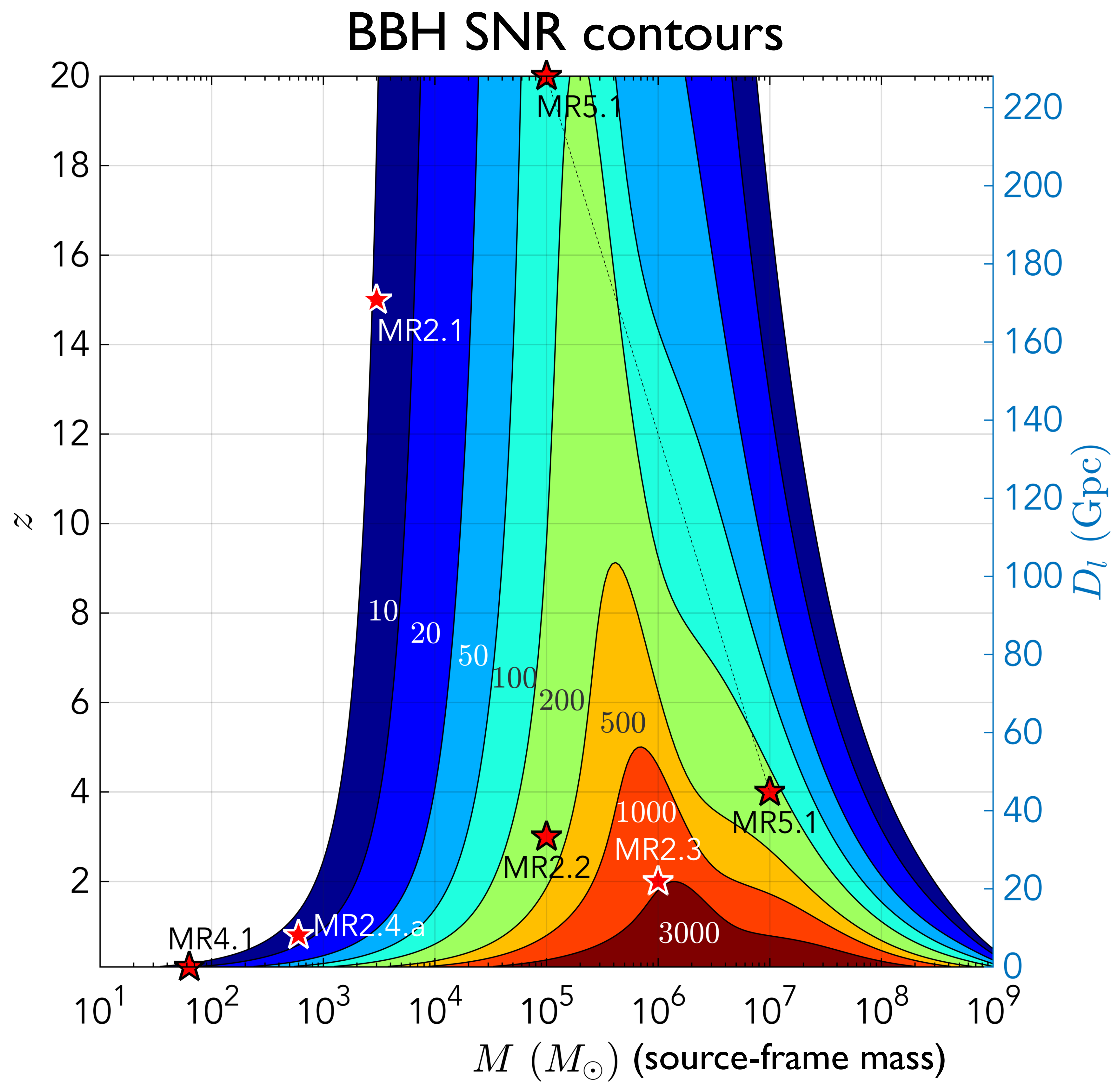
Sylvain Marsat (APC, Paris)

in collaboration with J. Baker (NASA GSFC), T. Dal Canton (LAL), S. Babak (APC), A. Toubiana (APC), M. Katz (AEI)

[Marsat&Baker arXiv/1806.10734]
[Marsat, Baker, Dal Canton arXiv/2003.00357]

LISA MBHB science and motivation





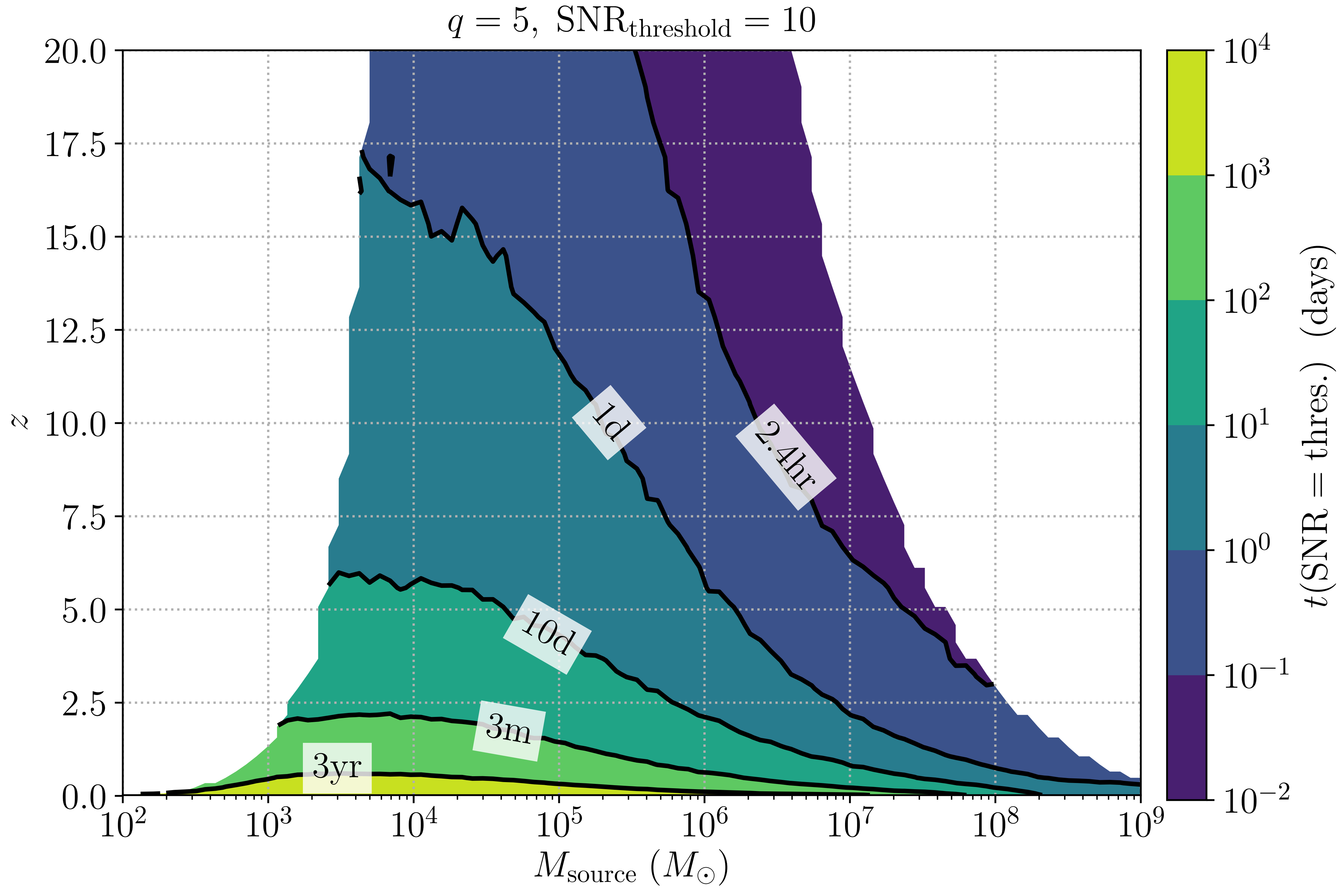
Motivation: what will LISA tell us ?

Massive black hole binaries (MBHB) are crucial sources for LISA.
How well can LISA extract their parameters ?

- Science questions: advance detection ? Localization for EM counterpart ? Cosmology ? Constrain population and formation channels ?
- Link to instrumental choices: tradeoffs
- Aim: Bayesian parameter estimation (PE) tools
- More realistic signals: include merger and higher harmonics
- Understand degeneracies

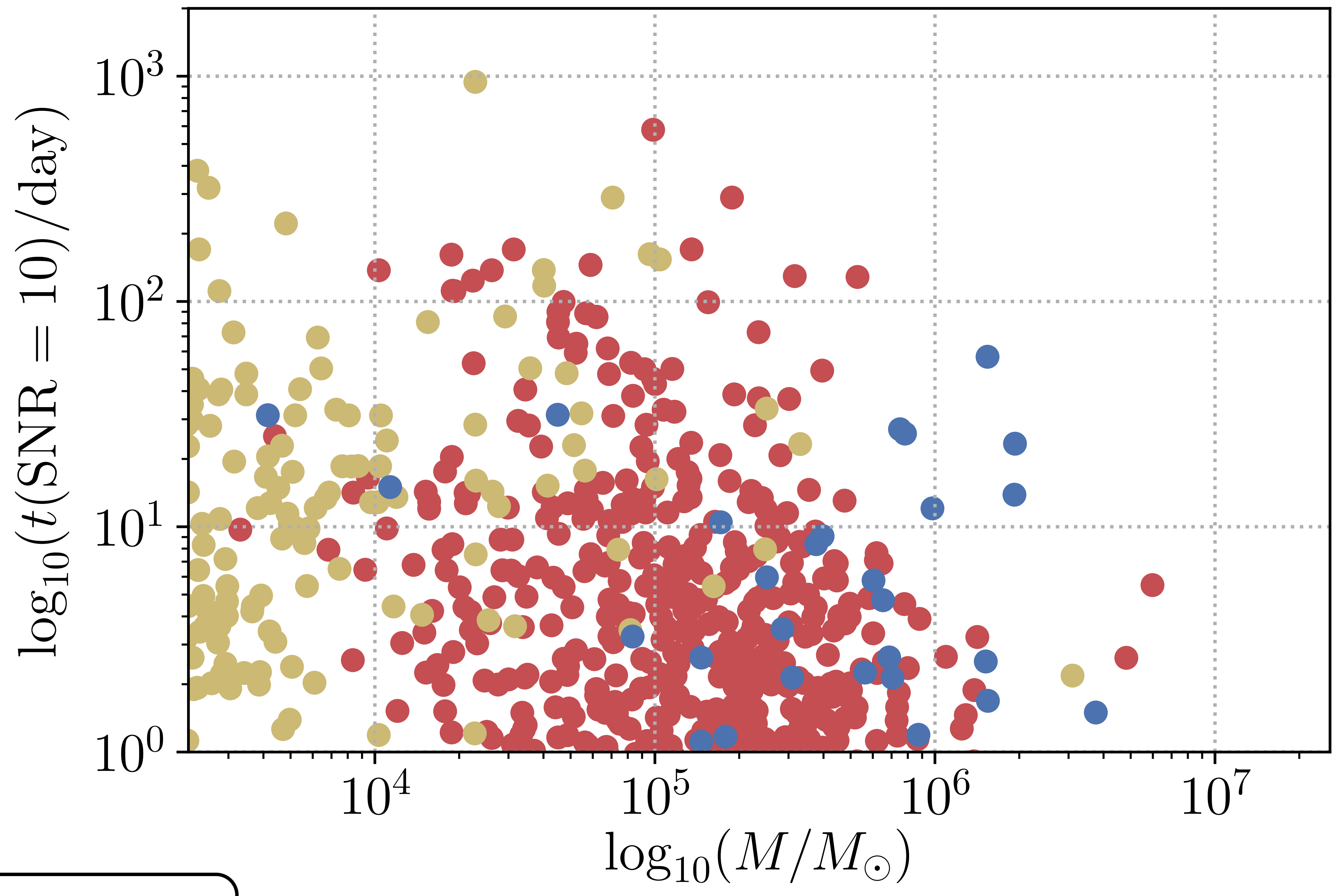
The length(s) of MBHB signals

- How long before merger can we detect the signal ?
- SNR=10 to claim detection



Astrophysical models [Barausse 2012]:

- Heavy seeds - delay
- Heavy seeds - no delay
- PopIII seeds - delay

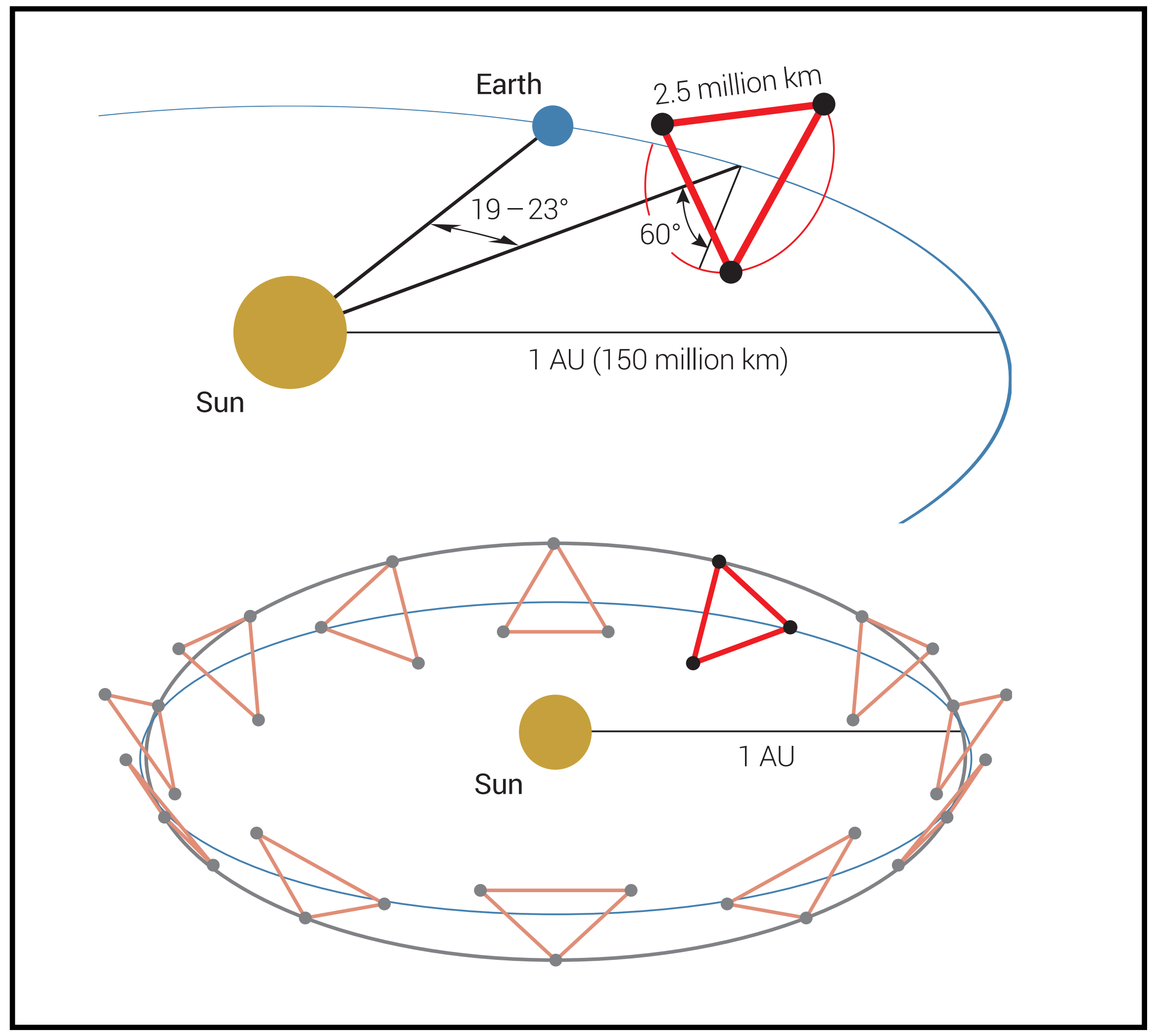


MBHB detected signals:
Bulk shorter than ~10days
Tail extending to ~3months

LISA instrumental response

LISA orbits

SSB-frame: global view of the orbits



Response

From spacecraft s to spacecraft r
through link s: $y = \Delta\nu/\nu$

$$y_{slr} = \frac{1}{2}\frac{1}{1-\hat{k}\cdot n_l} n_l \cdot (h(t_s) - h(t_r)) \cdot n_l$$

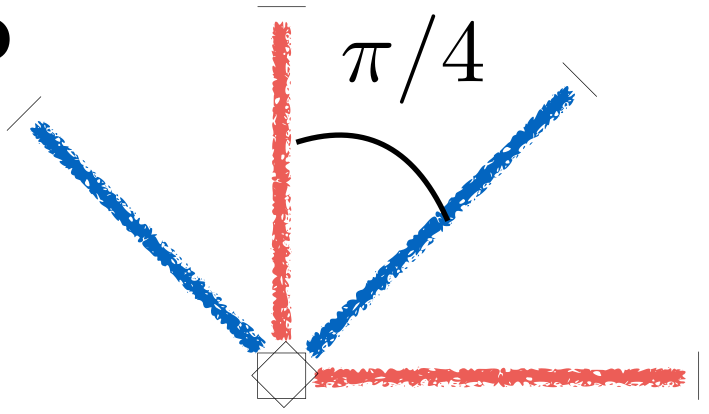
+ Time-delay interferometry (TDI)

Fourier-domain (separation of timescales [Marsat-Baker 2018])

$$\mathcal{T}_{slr} = \frac{i\pi f L}{2} \mathrm{sinc}\left[\pi f L \left(1 - k\cdot n_l\right)\right] \exp\left[i\pi f \left(L + k\cdot(p_r + p_s)\right)\right] n_l \cdot P \cdot n_l(t_f)$$

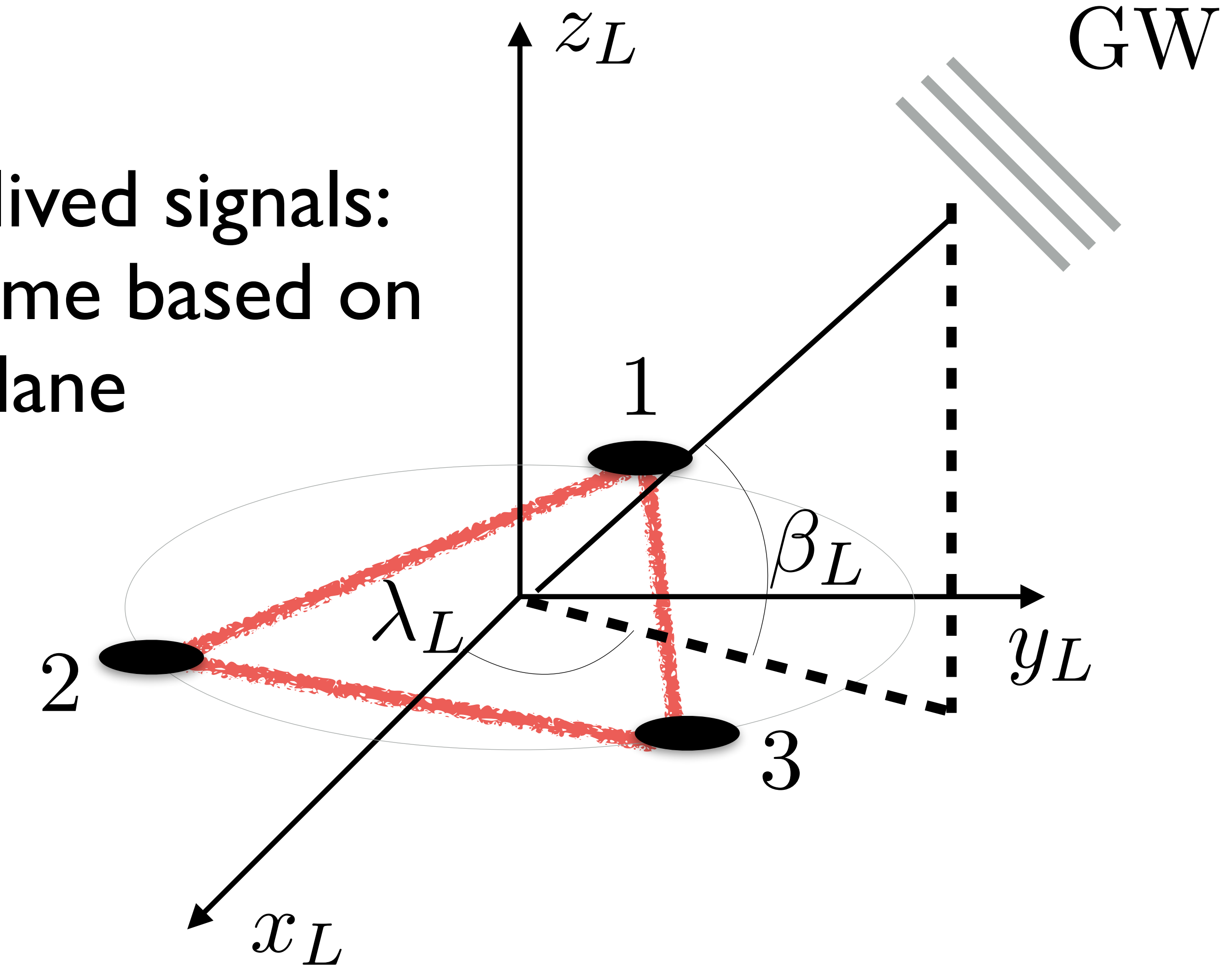
Time and frequency-dependency
Time: motion of LISA on its orbit
Frequency: departure from long-wavelength

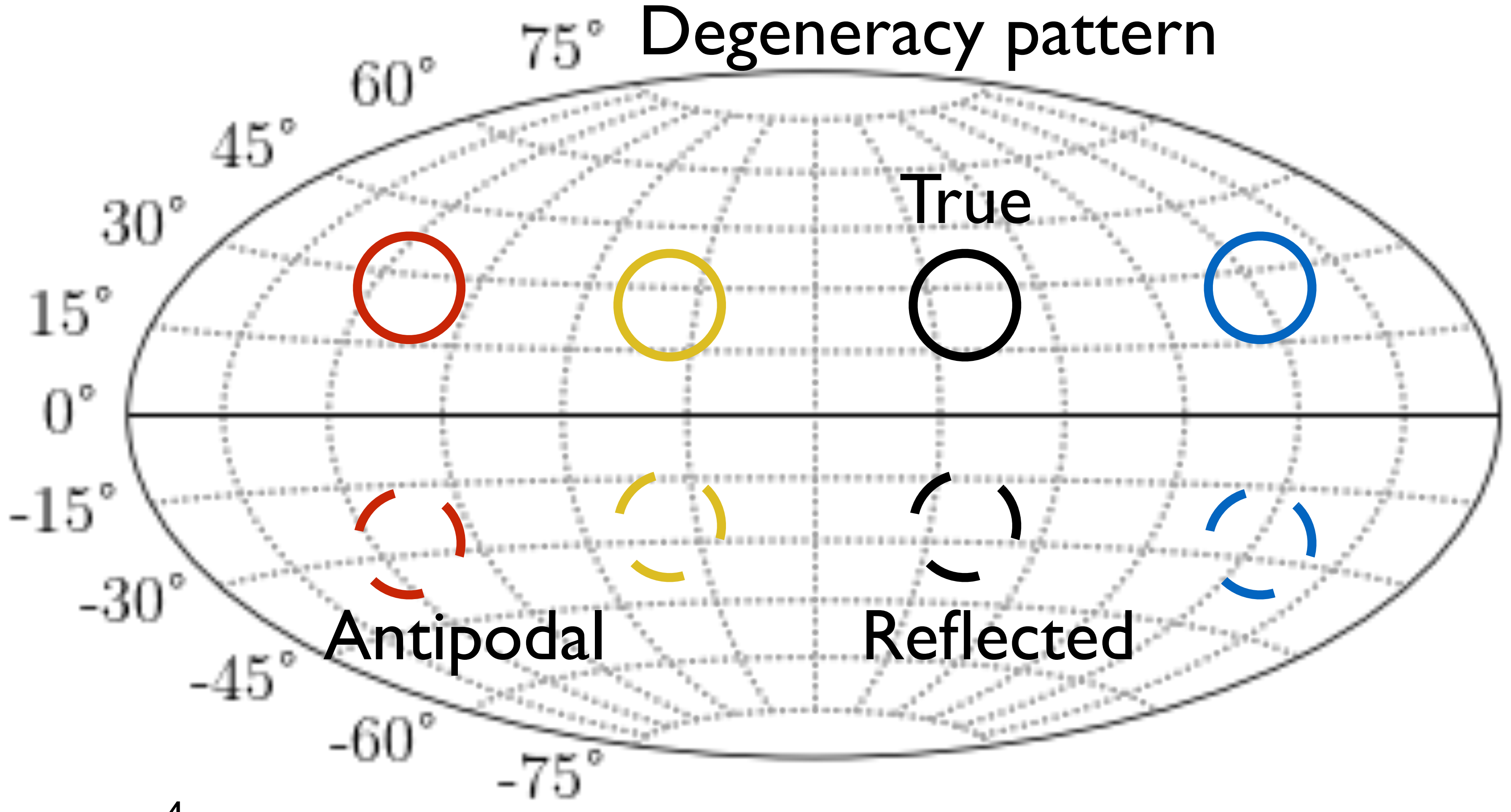
Low-f approximation: **two LIGO-type detectors** in motion [Cutler 1997]
High-f: more complicated



LISA frame

Short-lived signals: use frame based on LISA plane

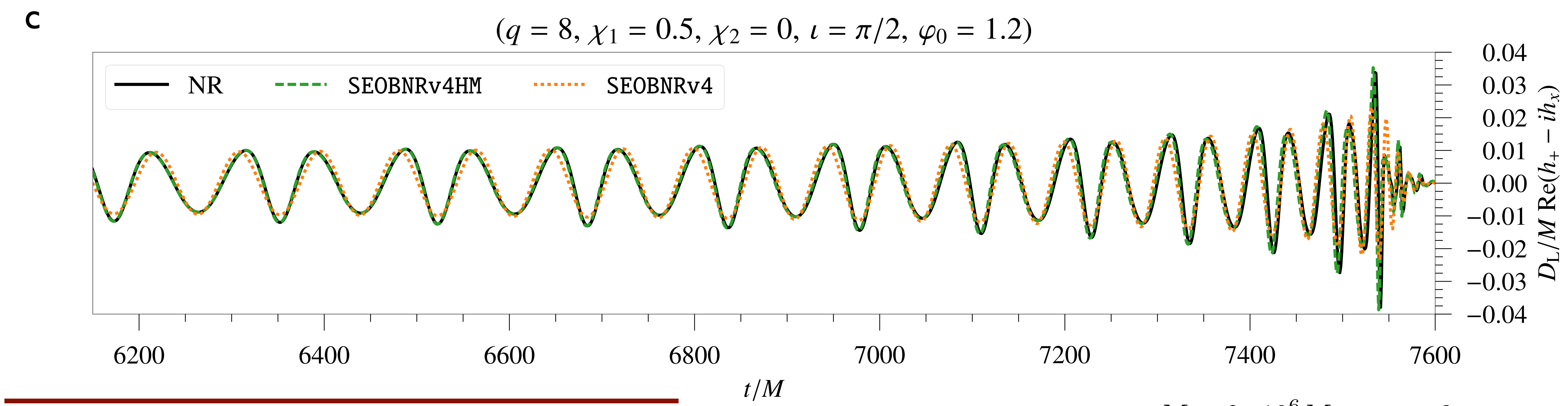




Higher harmonics in GW signals

Higher harmonics hlm

$$h_+ - i h_\times = \sum_{\ell \geq 2} \sum_{m=-\ell}^{\ell} {}_{-2}Y_{\ell m}(\iota, \varphi) h_{\ell m}$$

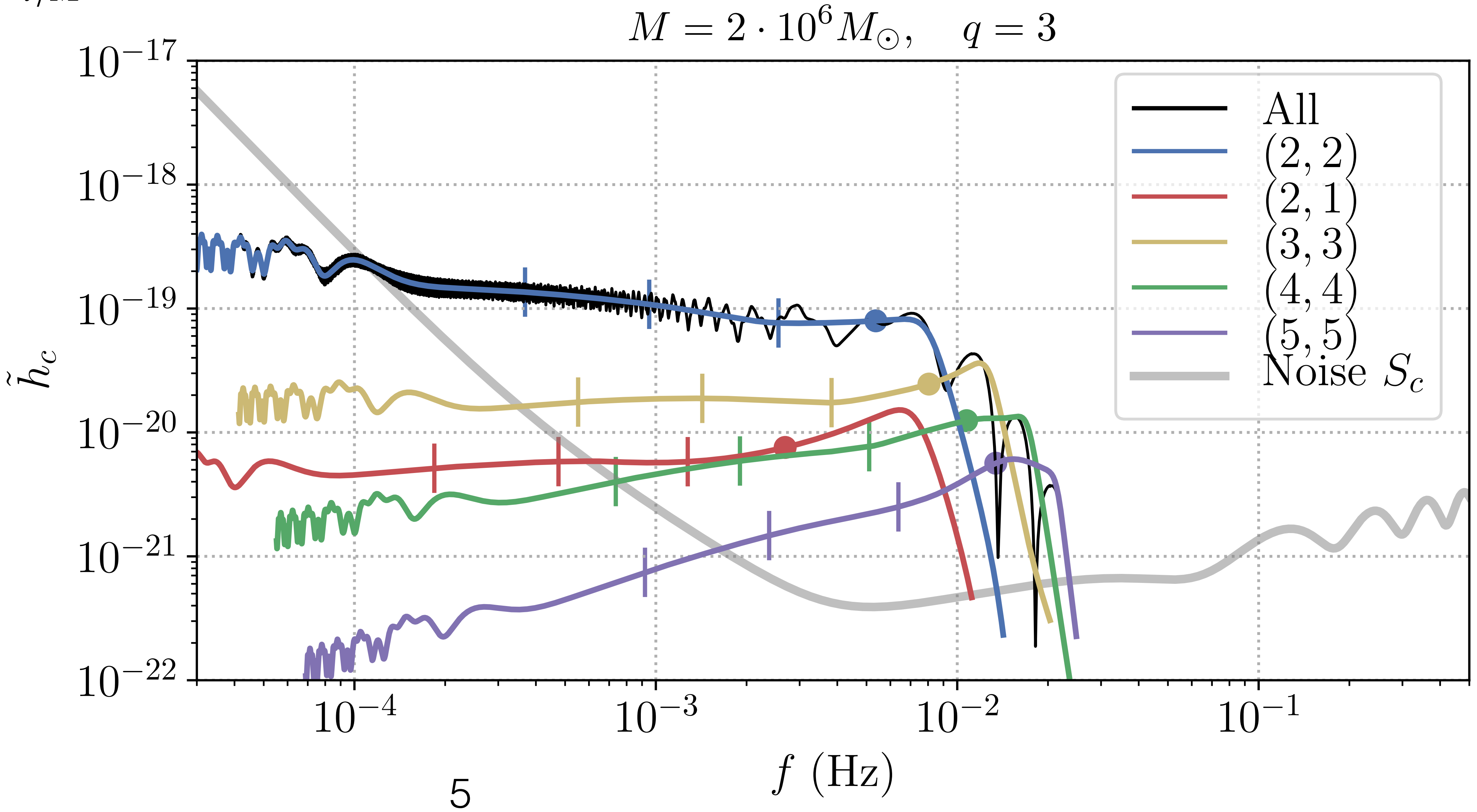


- Dominant harmonic h22
- Higher harmonics stronger at merger/ringdown
- Higher harmonics more important for high q and edge-on

MBHB with higher harmonics

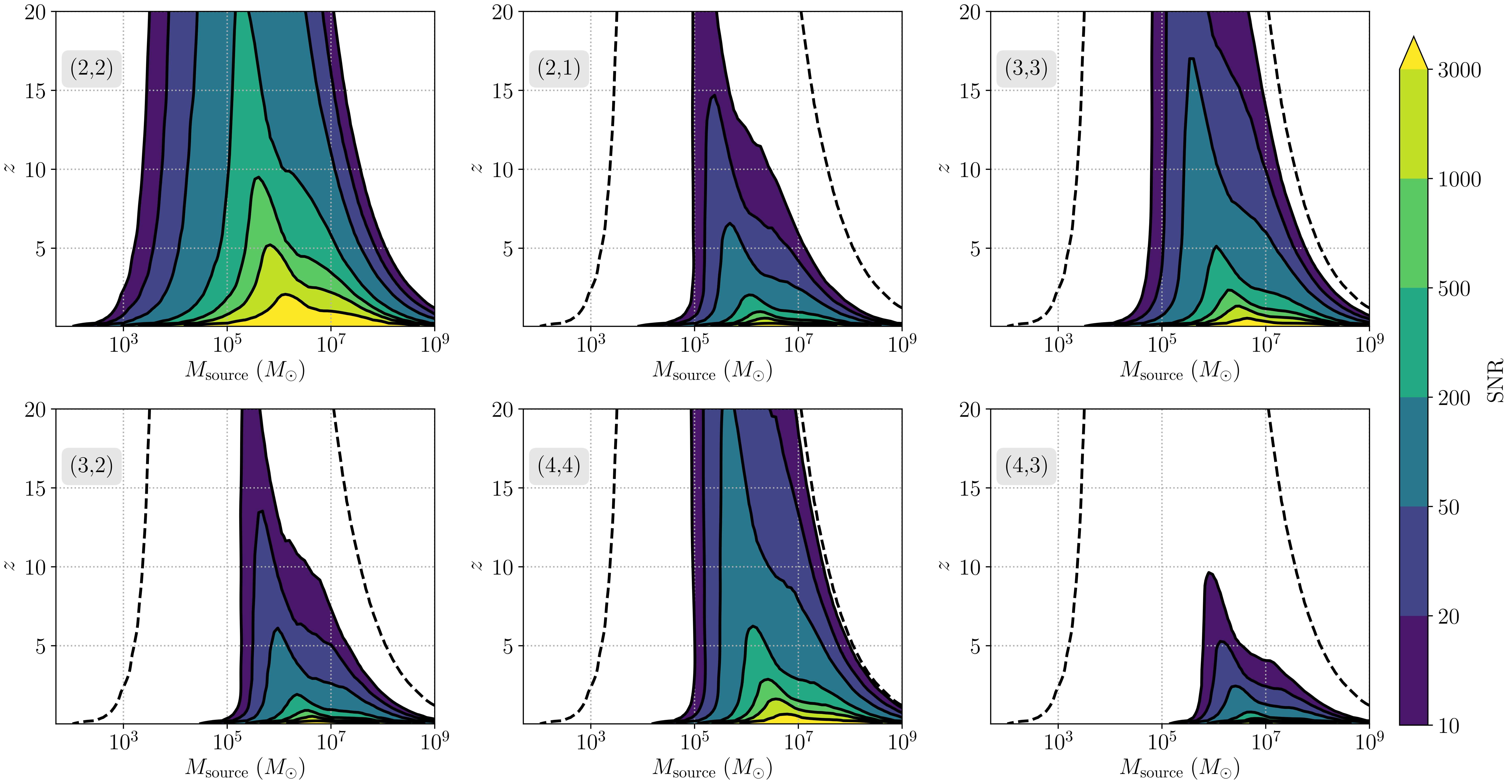
Ticks:

- SNR/64 (40h)
- SNR/16 (2.5h)
- SNR/4 (7min)
- merger



The SNR of higher harmonics

PhenomHM, q=5, averaging over spins and orientations



[Preliminary]

'Mode SNR':

$$\sqrt{(s_{\ell m}|s_{\ell m})}$$

Cross-terms not negligible

$$\sum_{(\ell m)\neq(\ell' m')} (s_{\ell m}|s_{\ell' m'})$$

Bayesian methodology

Bayesian analysis

Posterior: $p(\theta|d) = \dfrac{\mathcal{L}(d|\theta) p_0(\theta)}{p(d)}$

Likelihood: $\ln \mathcal{L}(d|\theta) = - \sum_{\text{channels}} \frac{1}{2} (h(\theta) - d | h(\theta) - d)$

Data: signal+noise $d = s + n$

- PE example: 0-noise simulation, n=0
- LDC: noise included

Samplers:

- MultiNest: Nested Sampling
- Parallel Tempering MCMC, differential evolution

Millions of likelihoods needed

Computational performance of waveforms/likelihoods crucial

Waveforms

$$h_+ - i h_\times = \sum_{\ell,m} {}_{-2}Y_{\ell m}(\iota, \varphi) h_{\ell m}$$

- PE example: EOBNRv2HM (Non-spinning model, includes modes (22, 21, 33, 44, 55) + Reduced Order Model (ROM) for sub-ms evaluation
- LDC: PhenomD (22, spins aligned)
- Fast Fourier-domain LISA response [Marsat-Baker 2018]

Accelerated likelihoods

$$(h_1|h_2) = 4\mathrm{Re} \int df \, \frac{\tilde{h}_1(f) \tilde{h}_2^*(f)}{S_n(f)}$$

Likelihood for n=0:

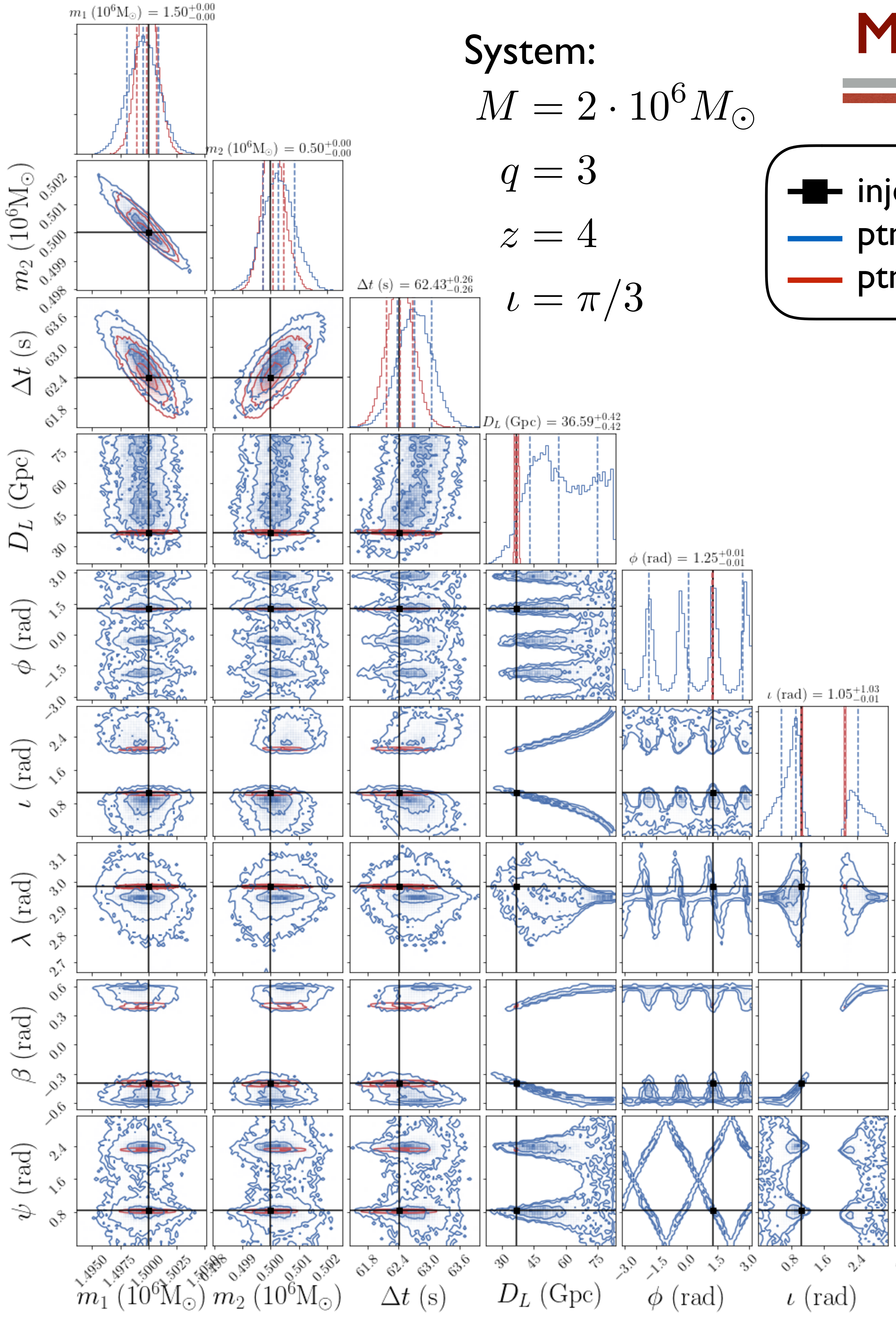
- Sparse grids: amplitude/phase and response
- Cubic spline representation 300-800 pts
- **Cost: h22: 1-2ms, 5 modes hlm: 15ms**

Likelihood with noise:

- Downsample for a short MBHB signal
- **Cost: h22: 3-5ms**

+ GPU acceleration: [Katz&al 2020]

MBHB PE 22 vs HM: degenerate case

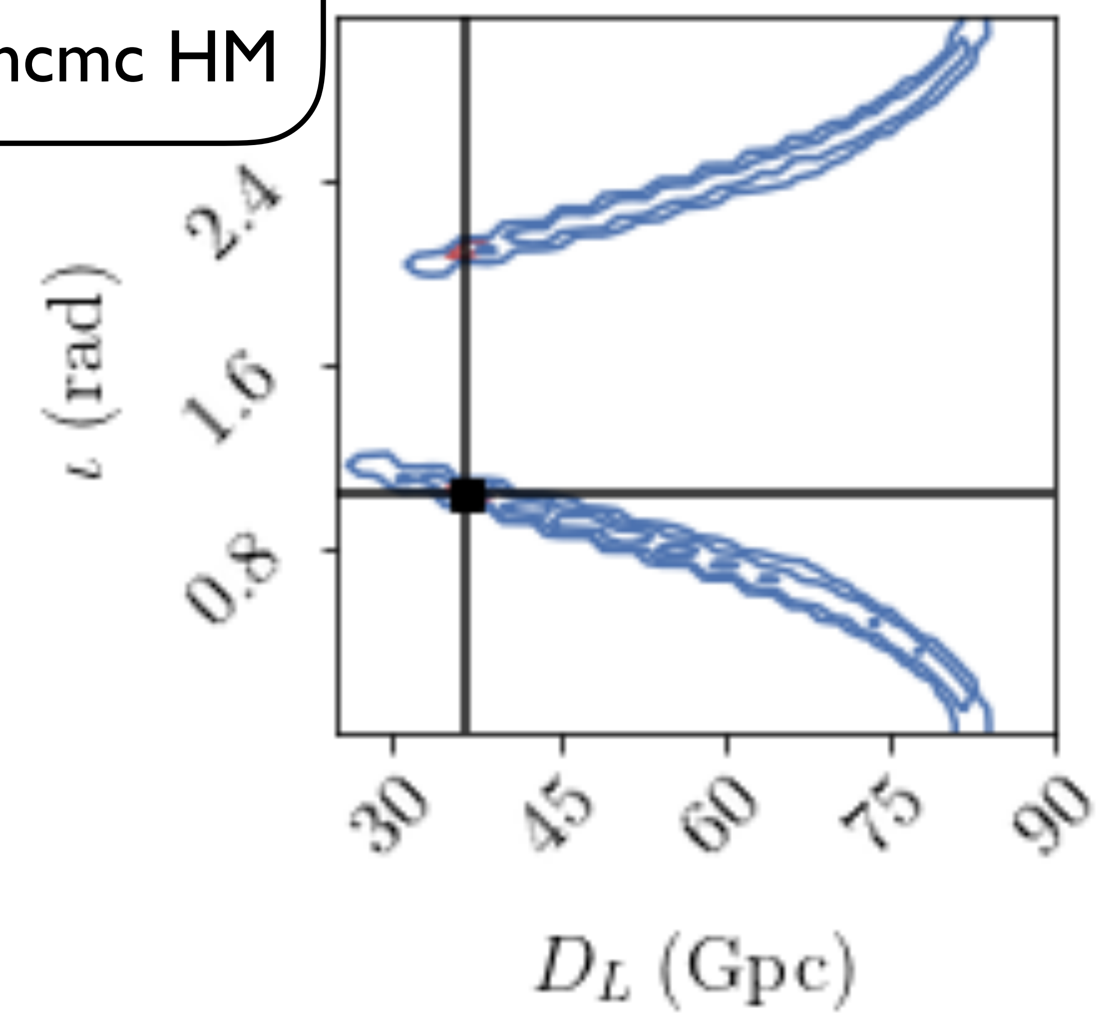


System:

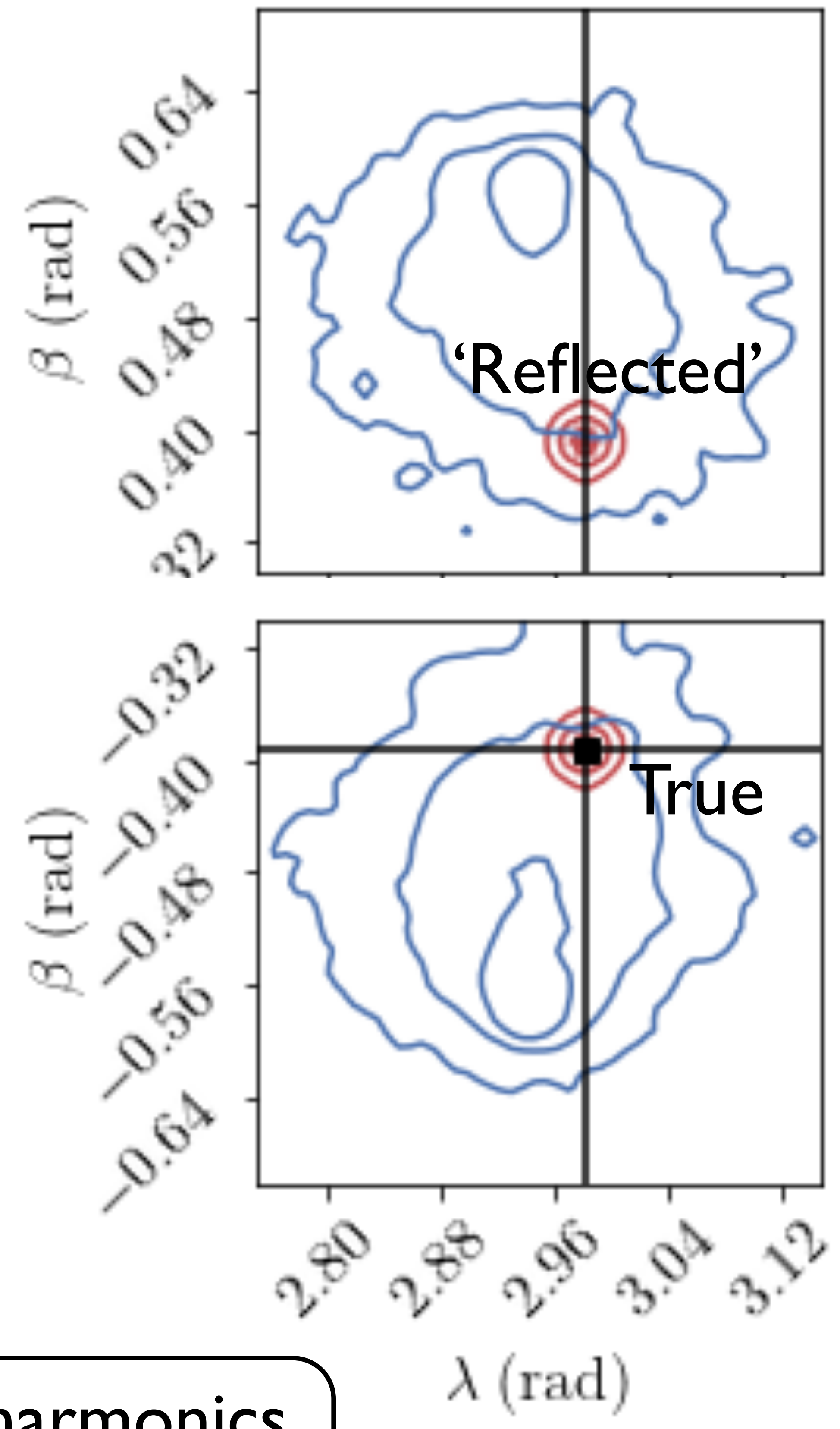
$$M = 2 \cdot 10^6 M_\odot$$
$$q = 3$$
$$z = 4$$
$$\iota = \pi/3$$

injection
ptmcmc 22
ptmcmc HM

Distance-inclination



Sky position



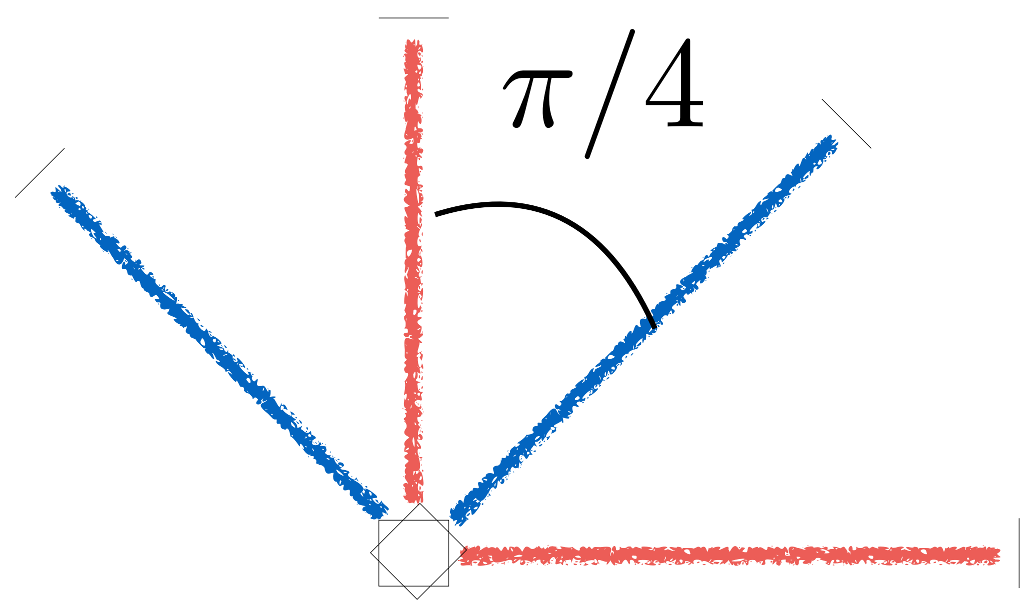
Higher harmonics crucial

Exploring degeneracies: simplified extrinsic likelihood

Simplified extrinsic response

Relevant for massive systems

- Neglect all LISA motion for the duration of the signal
- Low-frequency response
- Fix masses, time, and vary extrinsic parameters only



Equivalent to 2 immobile colocated LIGOs

Analytic likelihood function

$$\ln \mathcal{L} = -\frac{1}{2}\Lambda \left(\left| s_a - s_a^{\rm inj} \right|^2 + \left| s_e - s_e^{\rm inj} \right|^2 \right)$$

Λ precomputed normalization constant

Purely analytic response:

$$s^{22}_{a,e} = \frac{1}{4d}\sqrt{\frac{5}{\pi}} \cos^4 \frac{\iota}{2} e^{2i(\varphi - \psi_L)} \left(F^+_{a,e} + iF^\times_{a,e}\right) + \frac{1}{4d}\sqrt{\frac{5}{\pi}} \sin^4 \frac{\iota}{2} e^{2i(\varphi + \psi_L)} \left(F^+_{a,e} - iF^\times_{a,e}\right)$$

$F^+_{a,e}, F^\times_{a,e}$ pattern functions for channels A,E

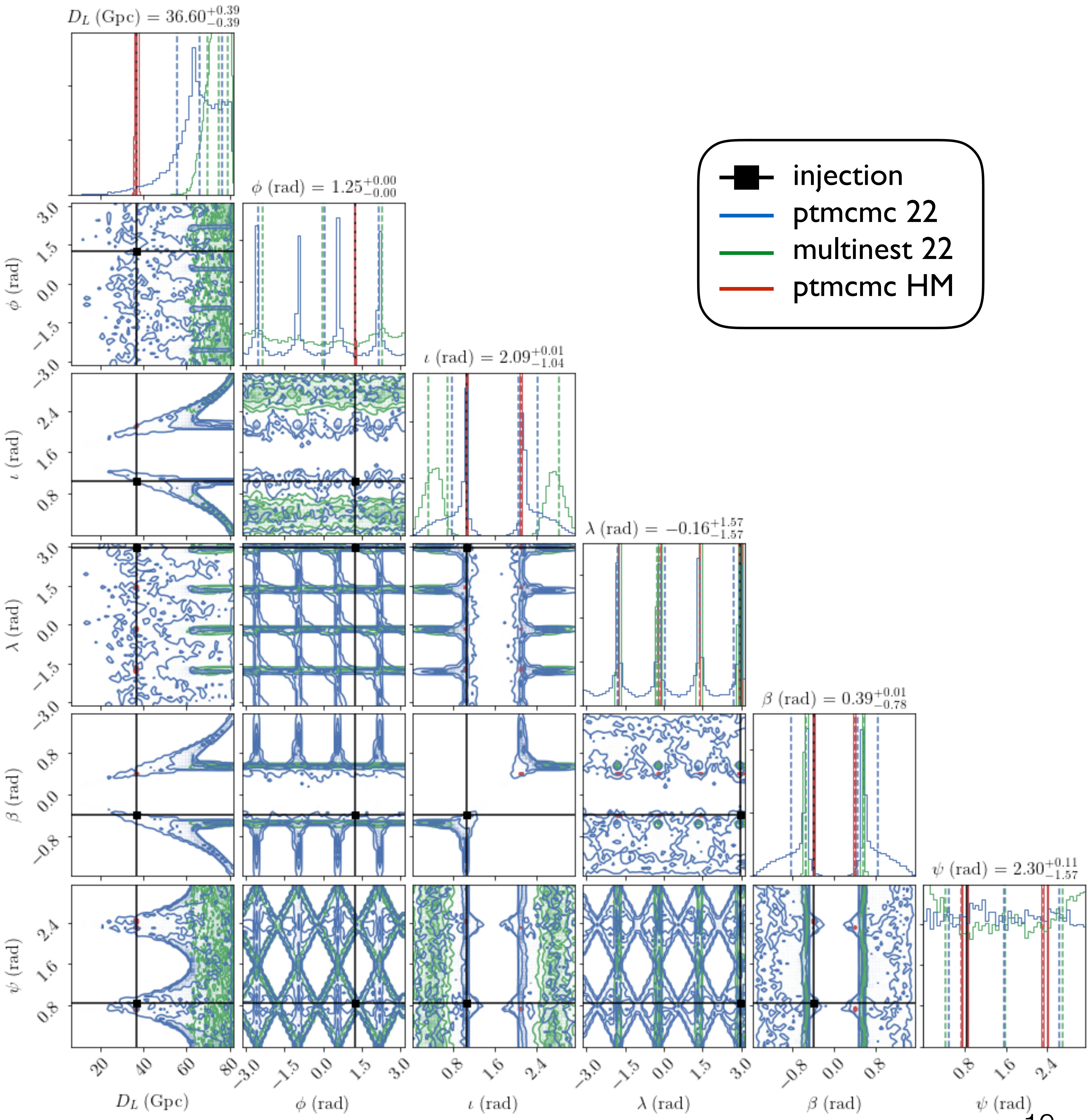
Predicting degeneracies

- Degeneracy distance/inclination, distance/latitude
- Special sky positions where a large volume of the multidimensional parameter space is allowed

$$\beta_L \approx \beta_L^\dagger = \pm \left(\frac{\pi}{2} - 2\text{Arctan } |r_{\rm inj}|^{1/4} \right) ,$$
$$\lambda_L \approx \lambda_L^\dagger = \frac{\pi}{6} - \frac{1}{4} \text{Arg } r_{\rm inj} \text{ mod } \frac{\pi}{2}$$

Degeneracy broken by higher harmonics (**caveat**: edge-on case)

Exploring degeneracies: simplified extrinsic likelihood



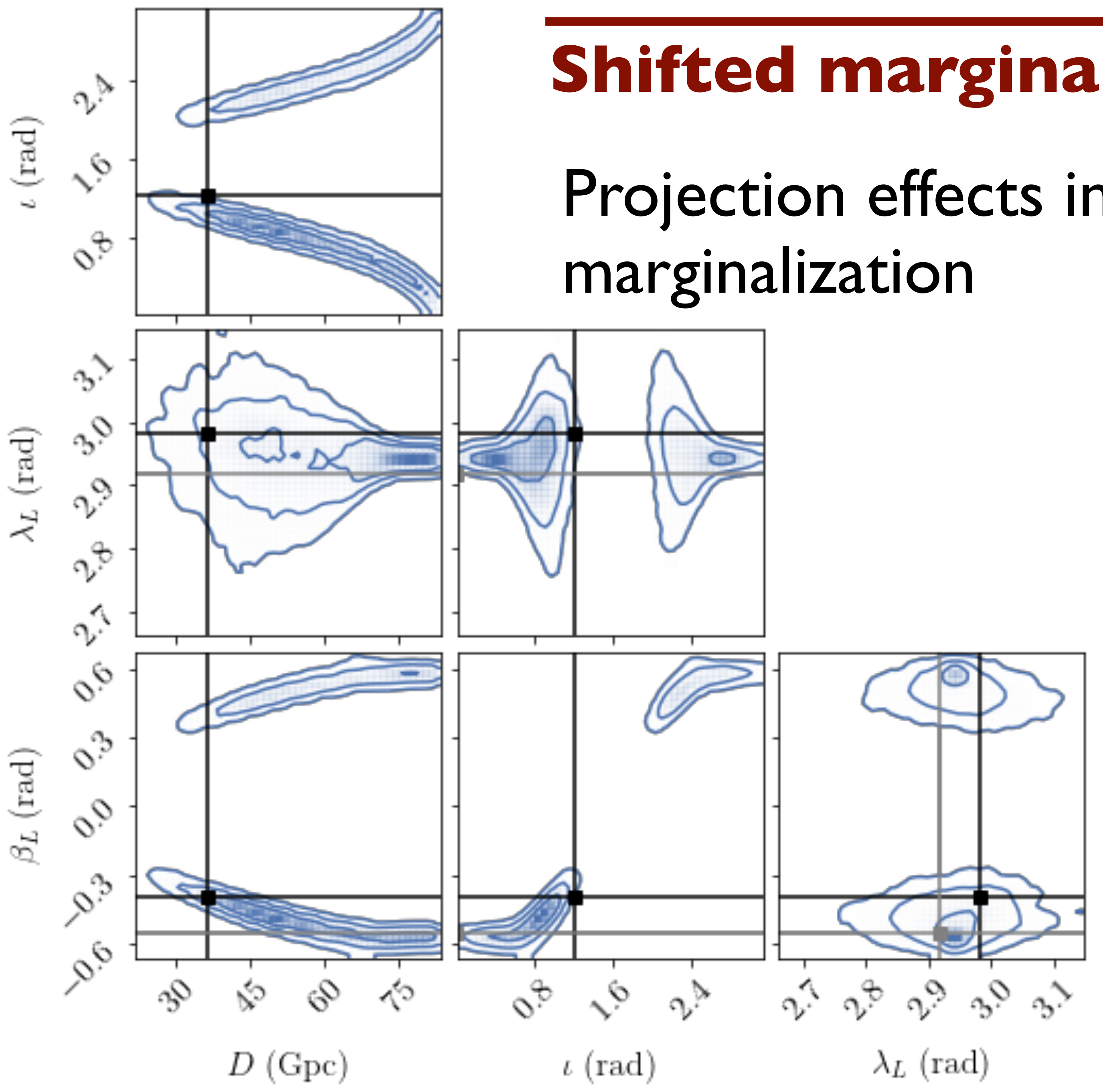
Simplified likelihood, extremely fast to evaluate:

$$\ln \mathcal{L} = -\frac{1}{2}\Lambda \left(\left| s_a - s_a^{\mathrm{inj}} \right|^2 + \left| s_e - s_e^{\mathrm{inj}} \right|^2 \right)$$

- Very extended degeneracies
- Multinest only captures a small region of the degenerate likelihood
- Extra symmetry inclination-latitude
- Qualitatively explains why degenerate regions at large distance-inclination and distance-latitude cause a shift in the sky

Useful benchmark for samplers
Worst-case degeneracy
Analytical description

Understanding degeneracy breaking by higher harmonics

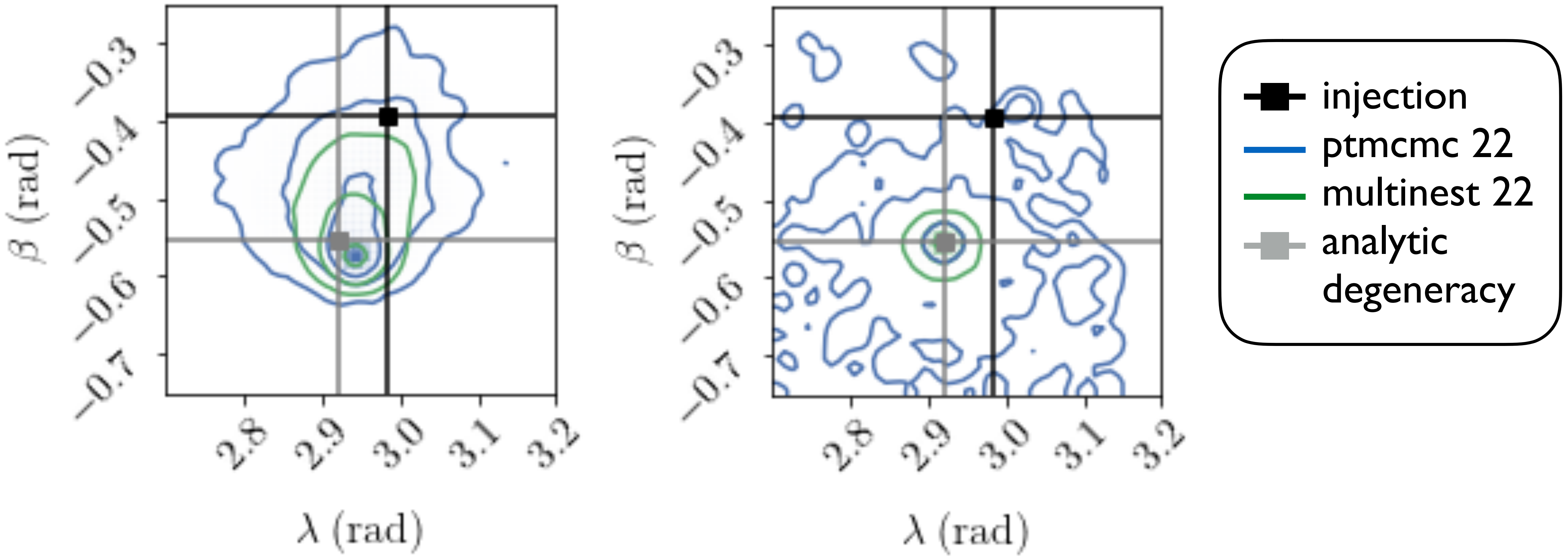


Shifted marginal sky posterior

Projection effects in marginalization

Sky zoom (h22), full response

Sky zoom (h22), no LISA motion and no high-f



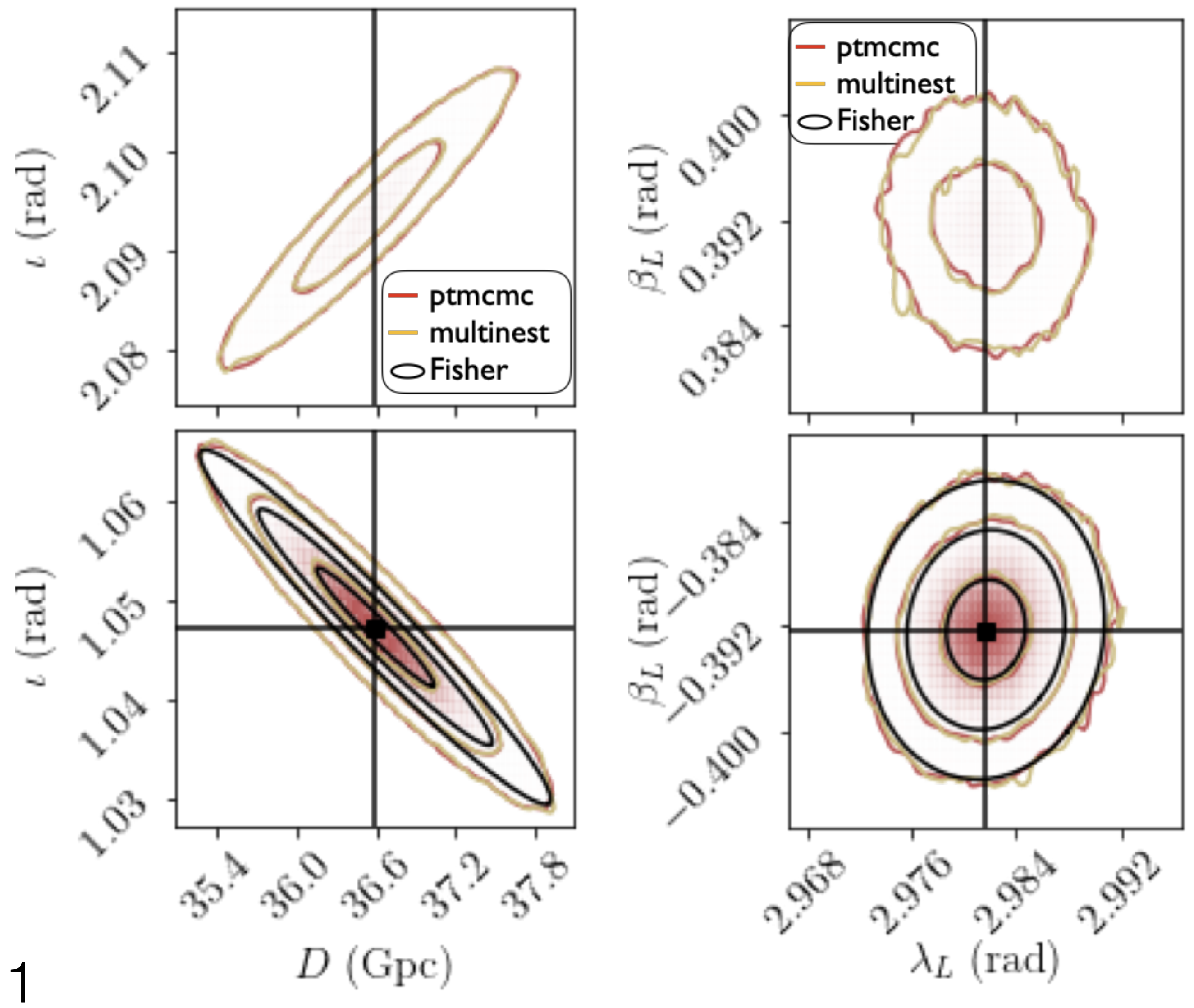
The role of higher harmonics

$$h_+ - ih_\times = \sum {}_{-2}Y_{\ell m}(\iota, \varphi) h_{\ell m}$$

$${}_{-2}Y_{\ell m}(\iota, \varphi) = {}_{-2}Y_{\ell m}(\iota, 0) e^{im\varphi}$$

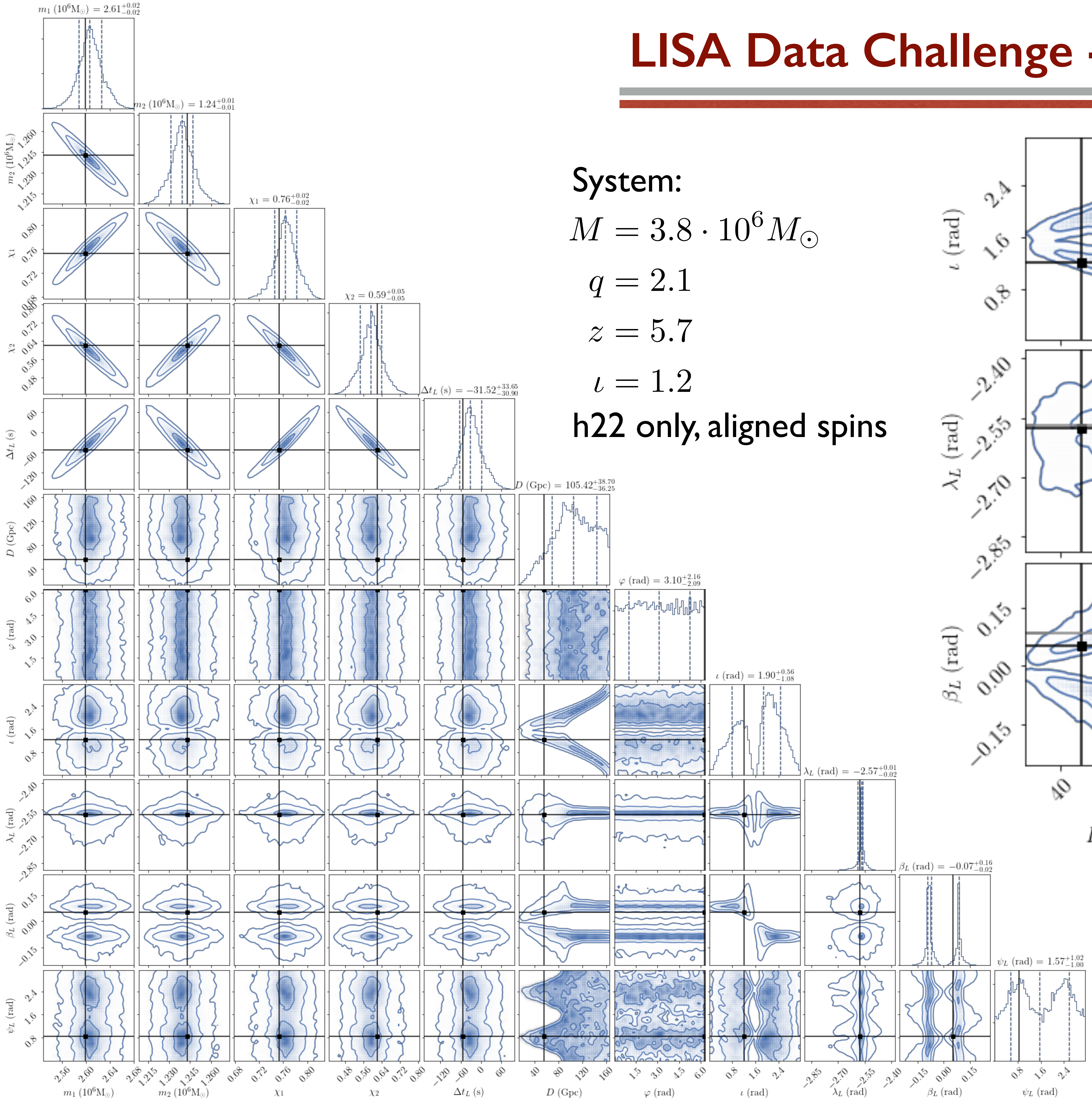
When measuring several modes $h_{\ell m}$:

- Distance/inclination degeneracy broken
- Phase independently measured
- Better sky localization (**caveat**: edge-on case)



Higher harmonics break degeneracies, still bimodal in sky

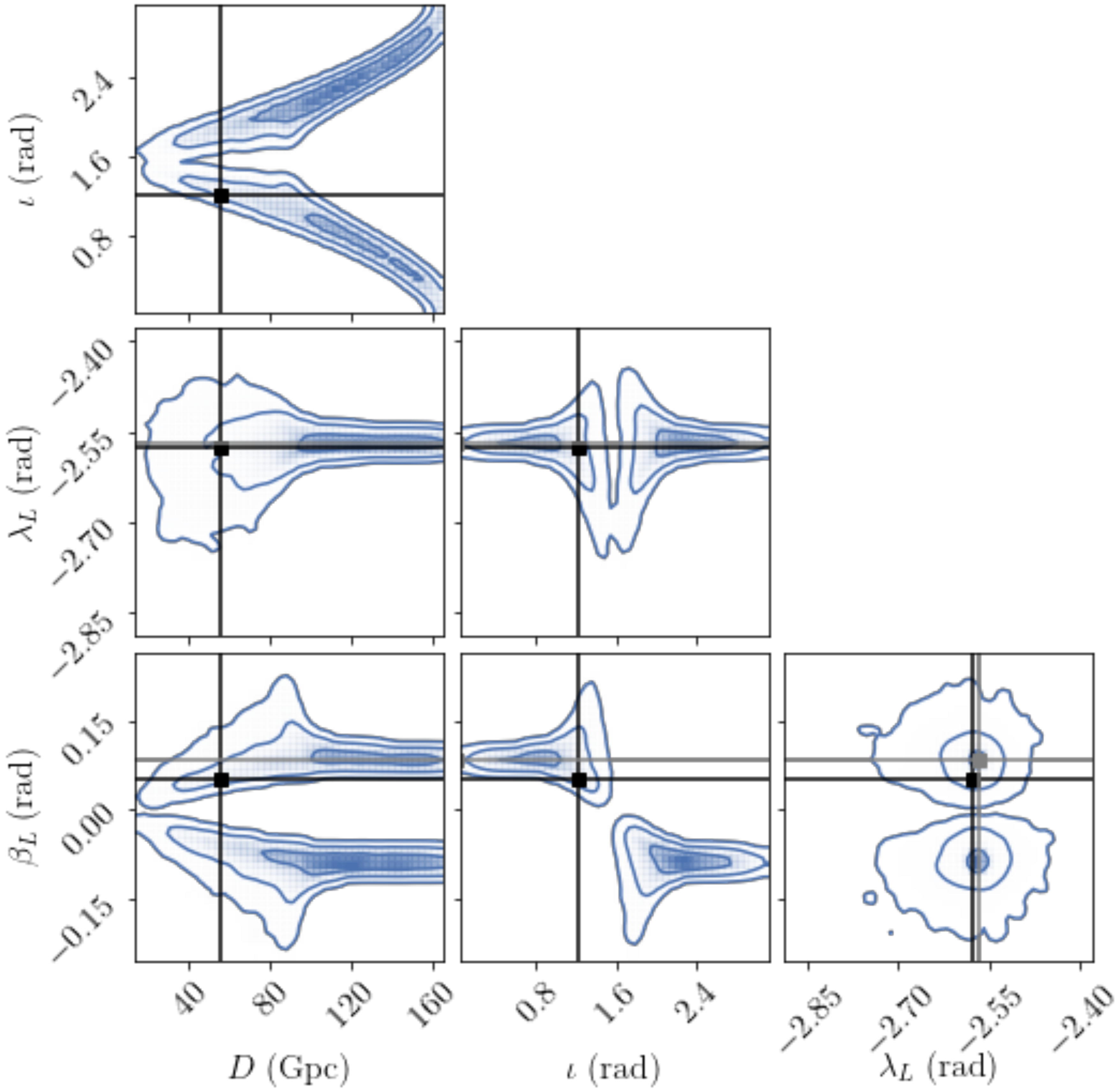
LISA Data Challenge - 1 MBHB results



System:

$$M = 3.8 \cdot 10^6 M_\odot$$
$$q = 2.1$$
$$z = 5.7$$
$$\iota = 1.2$$

h22 only, aligned spins



Pre-merger analysis: accumulation of information with time

Cutting signals before merger

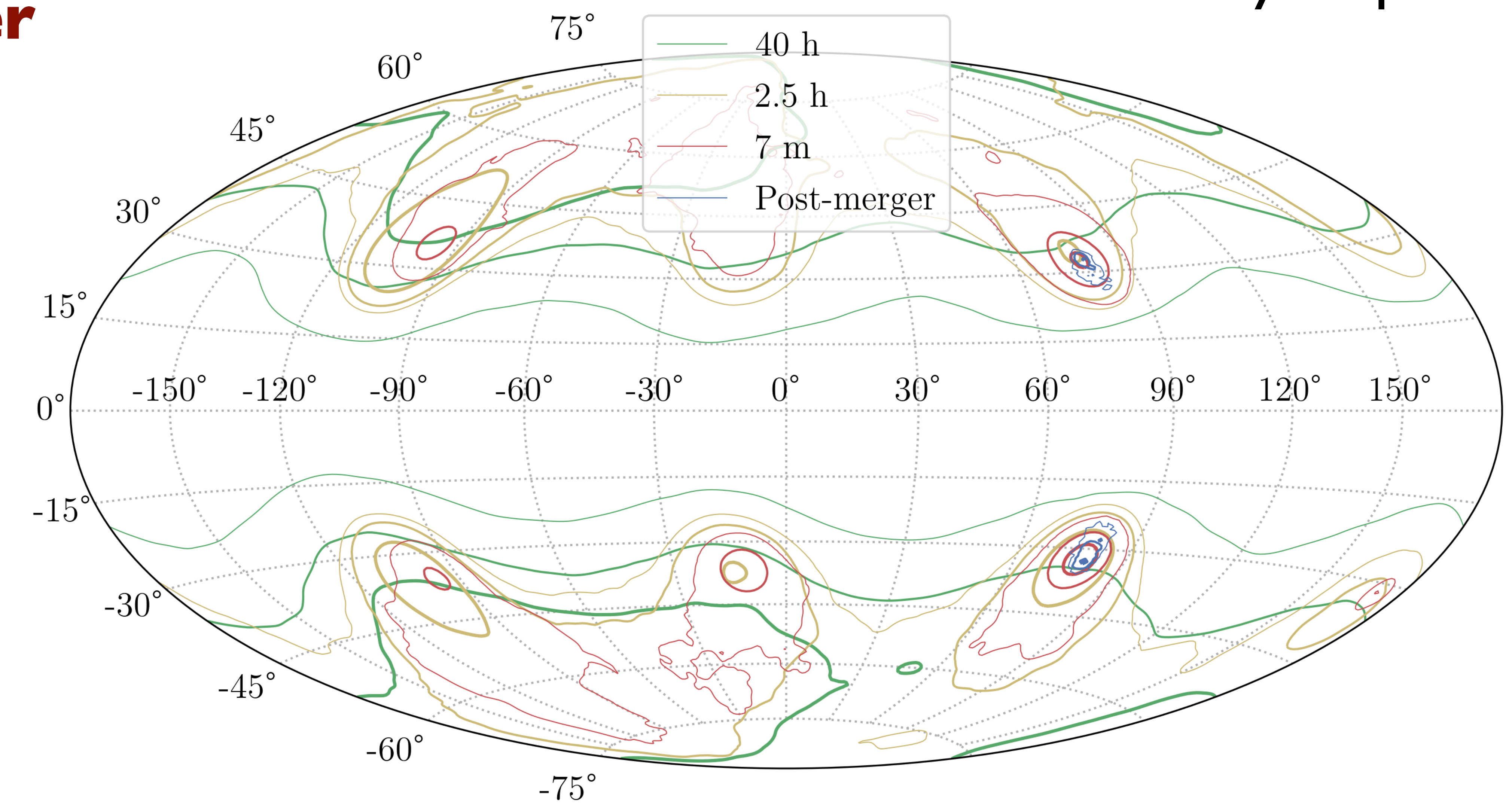
SNR-based cuts:

SNR	DeltaT
10	40h
42	2.5h
167	7min
666	-

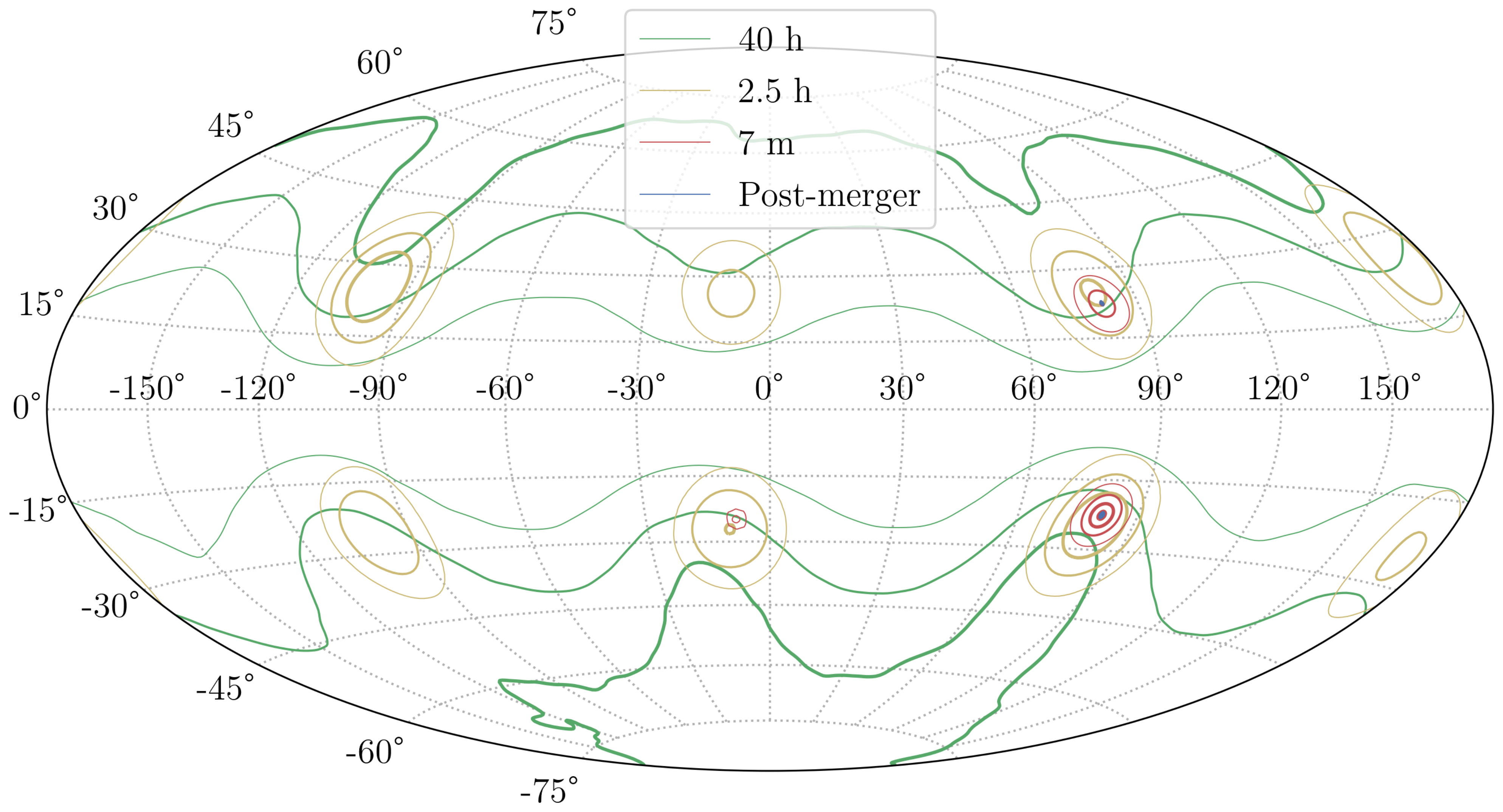
Note: z=4, short signal

Premerger localization (Fisher):
[Mangiagli&al 2020]

LISA-frame sky map 22



LISA-frame sky map hm



8-maxima sky degeneracy
only broken shortly before merger

2-maxima sky degeneracy
survives after merger ('Reflected')

Multimodal degeneracies
pre-merger
Not a golden source !

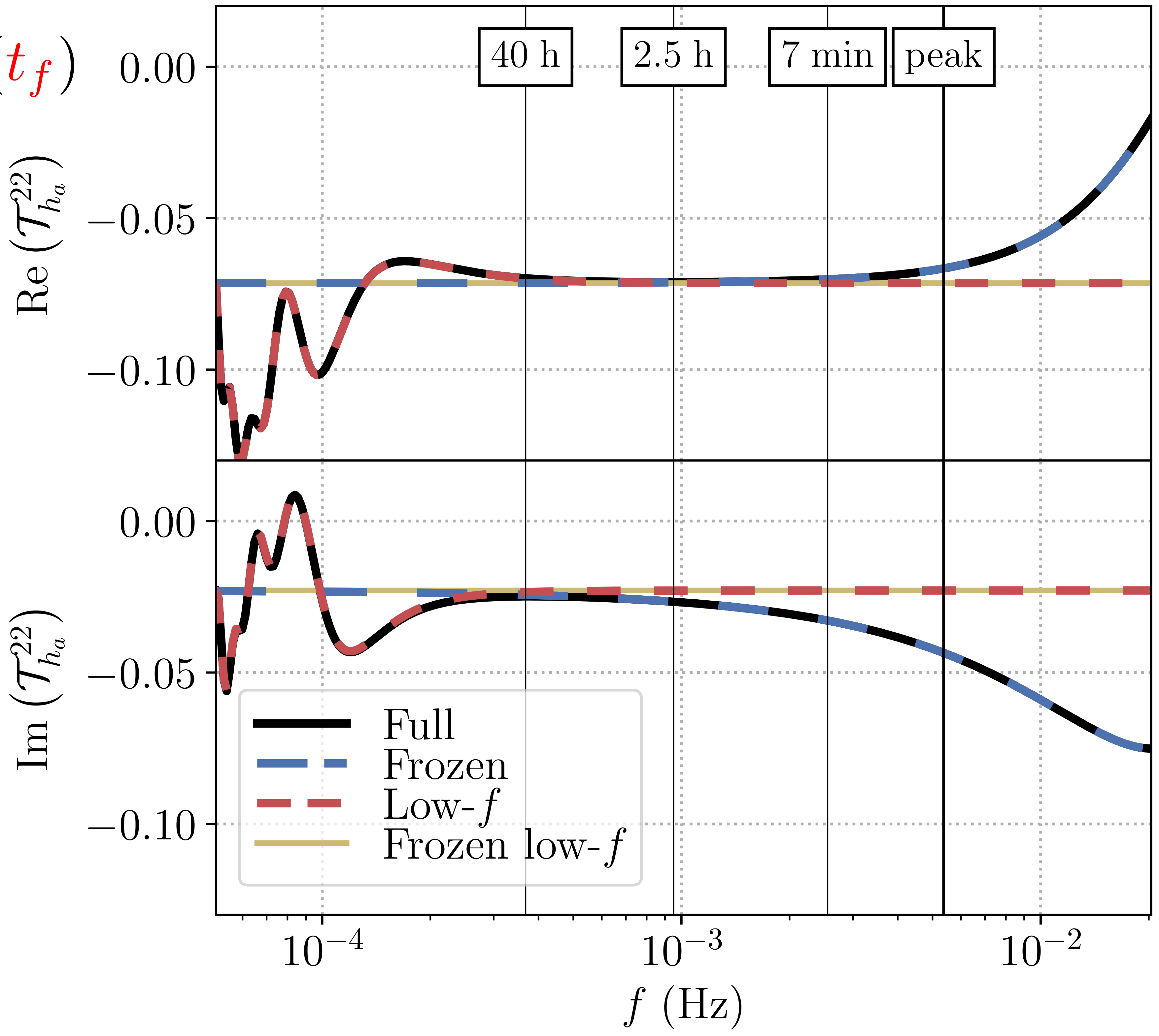
Pre-merger analysis: decomposing the instrument response

Decomposing the response

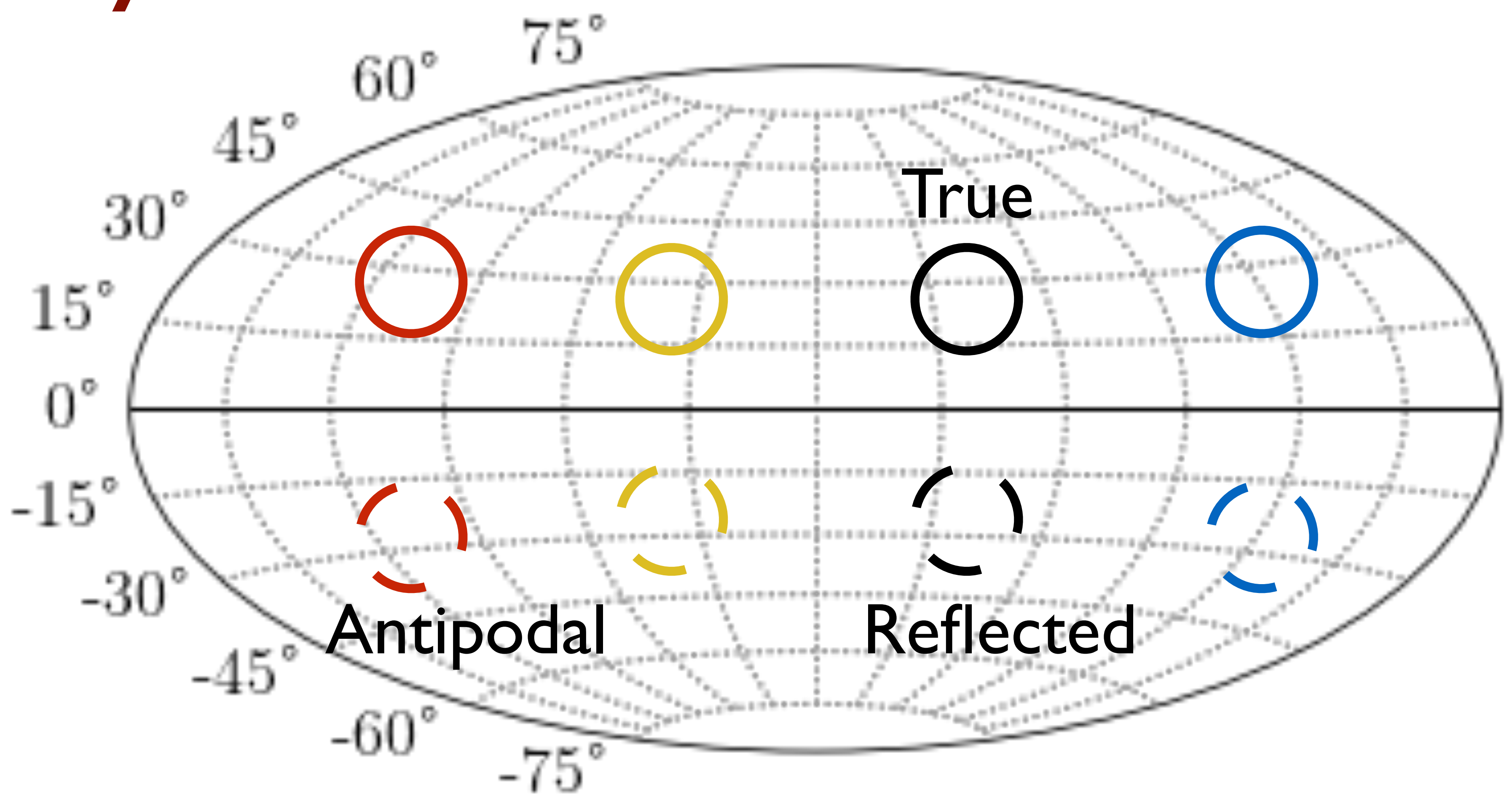
$$\mathcal{T}_{slr} = \frac{i\pi f L}{2}\mathrm{sinc}\left[\pi f L\left(1 - k\cdot n_l\right)\right]\exp\left[i\pi f\left(L + k\cdot(p_r+p_s)\right)\right] n_l\cdot P\cdot n_l(t_f)$$

Time and frequency-dependency in transfer functions
Time: motion of LISA on its orbit
Frequency: departure from long-wavelength approx.

Response:
- 'Full': keep all terms
- 'Frozen': ignore LISA motion
- 'Low-f': ignore f-dependency
- 'Frozen Low-f': ignore both



Sky modes L-frame



- 'True': true position
- 'Reflected': eliminated by motion, degenerate for high-f
- 'Antipodal': degenerate for motion, eliminated by high-f

Pre-merger analysis: likelihood with decomposed response

Degeneracy breaking for 8 sky maxima

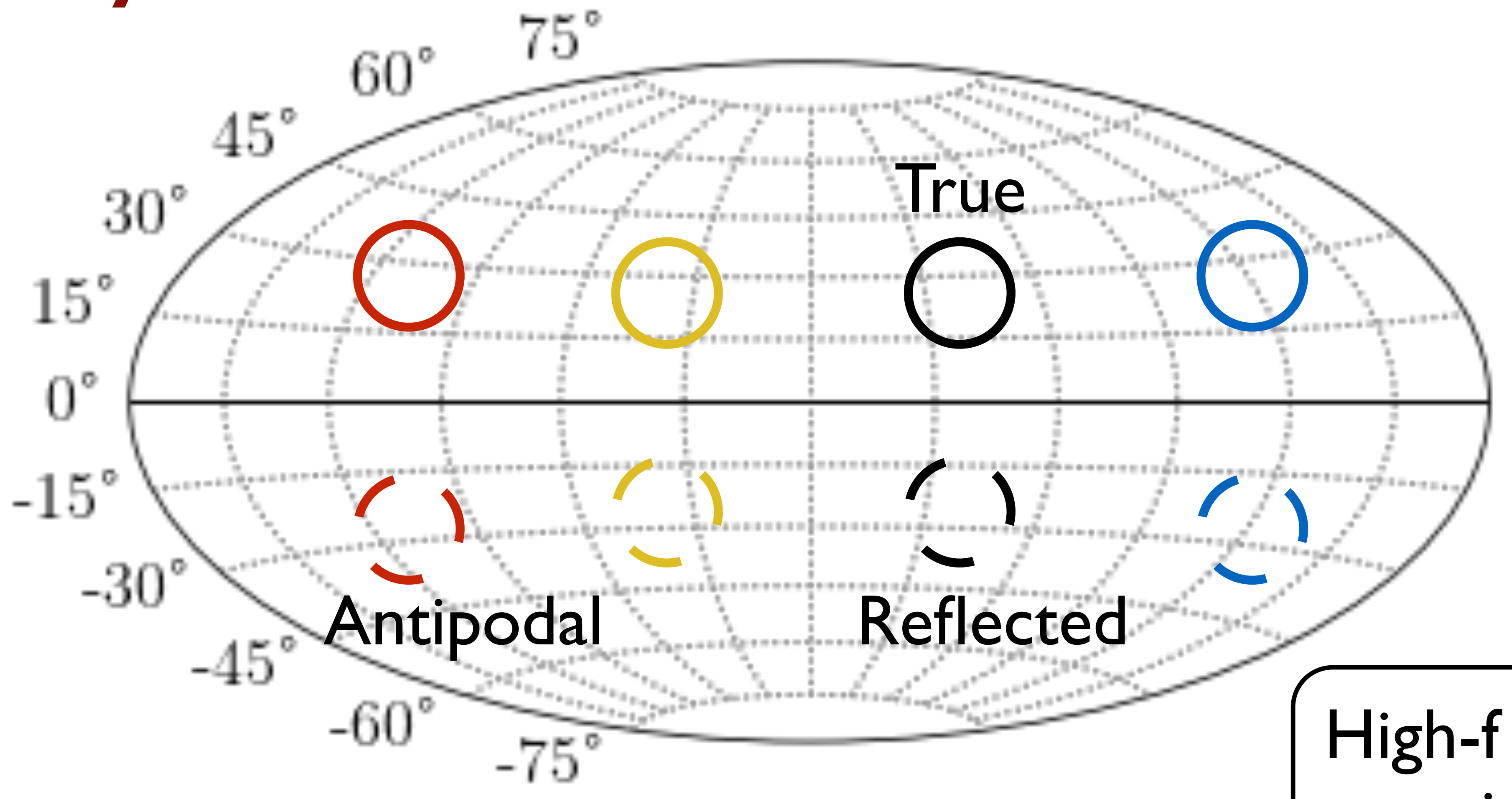
Instrument response:

- ‘Full’: keep all terms
- ‘Frozen’: ignore LISA motion
- ‘Low-f’: ignore f-dependency
- ‘Frozen Low-f’: ignore both

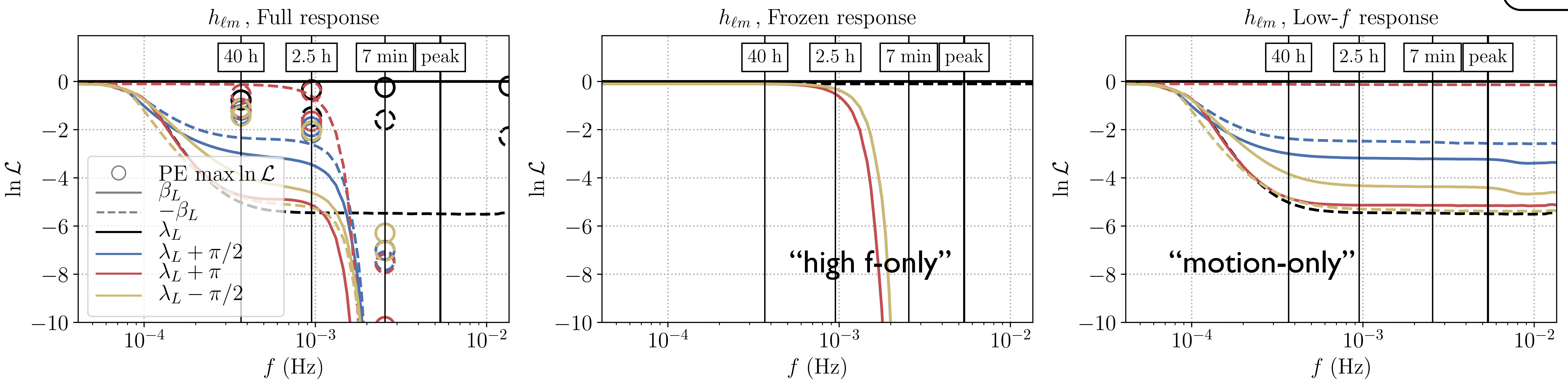
Likelihood: $\ln \mathcal{L}(d|\theta) = -\sum_{\text{channels}} \frac{1}{2}(h(\theta) - d | h(\theta) - d)$

Approximate degeneracy measure: likelihood at the other sky positions

Sky modes L-frame

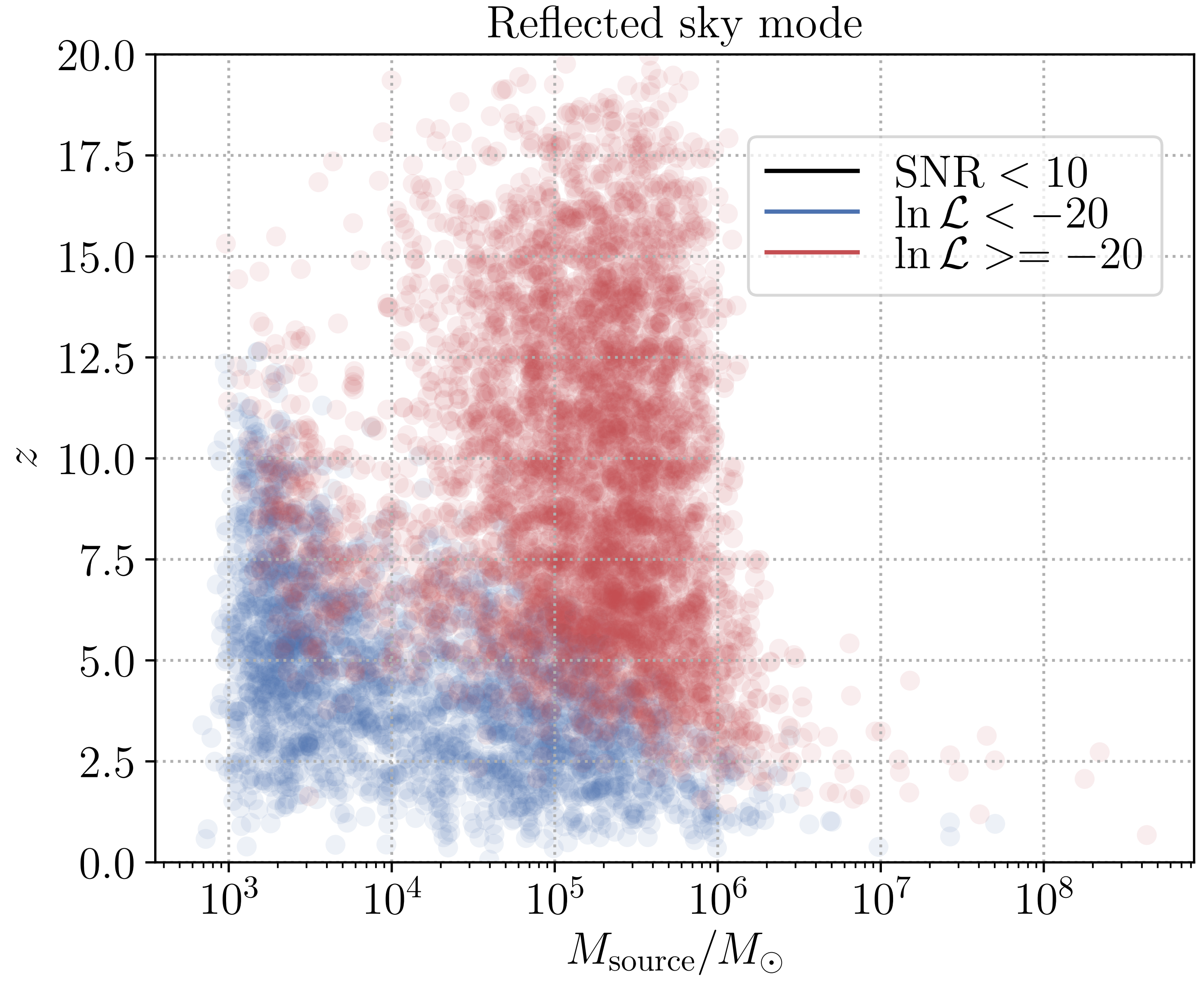


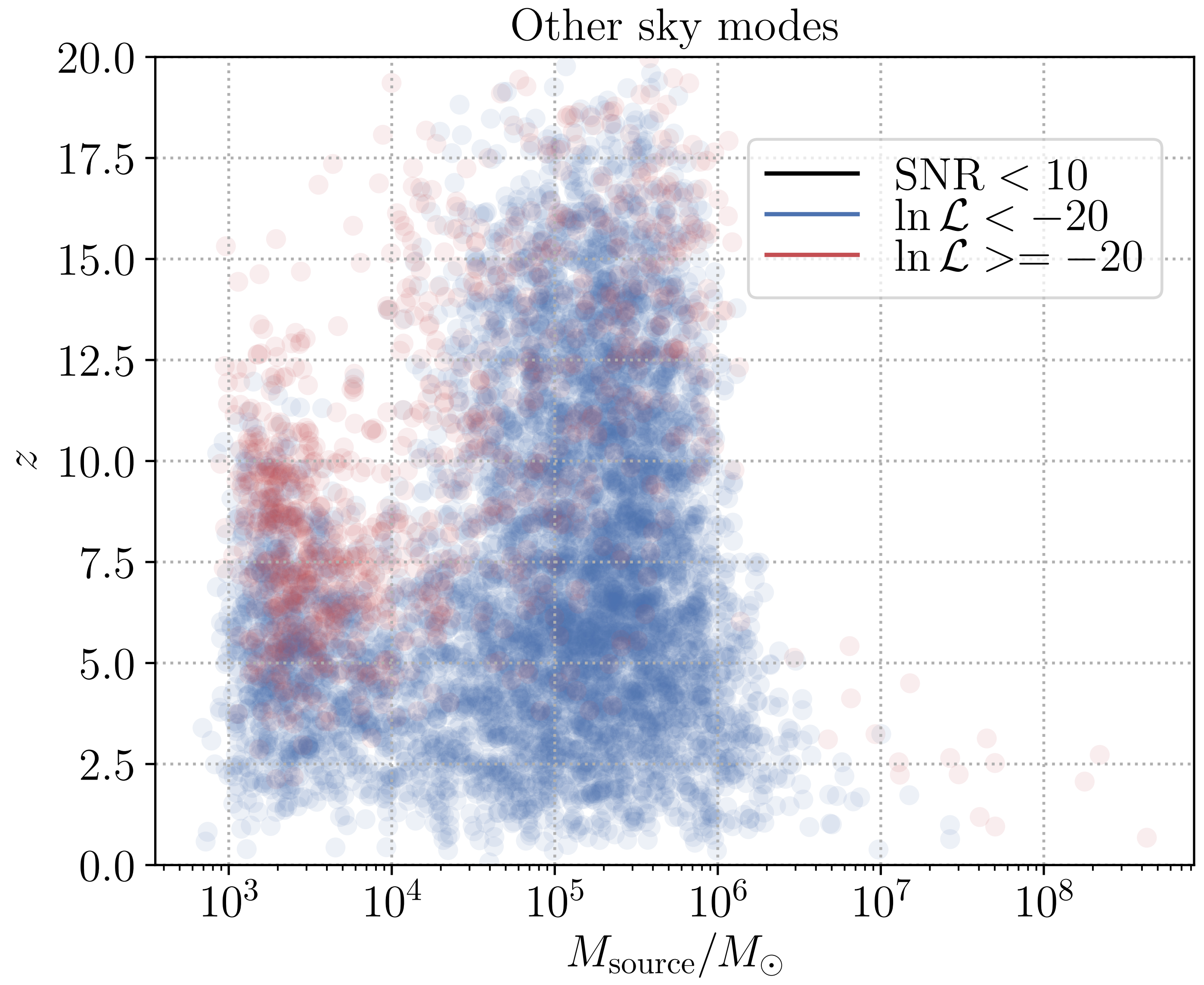
High-f features crucial in reducing multimodality (8->2)



Multimodality of the sky localization

Post-merger





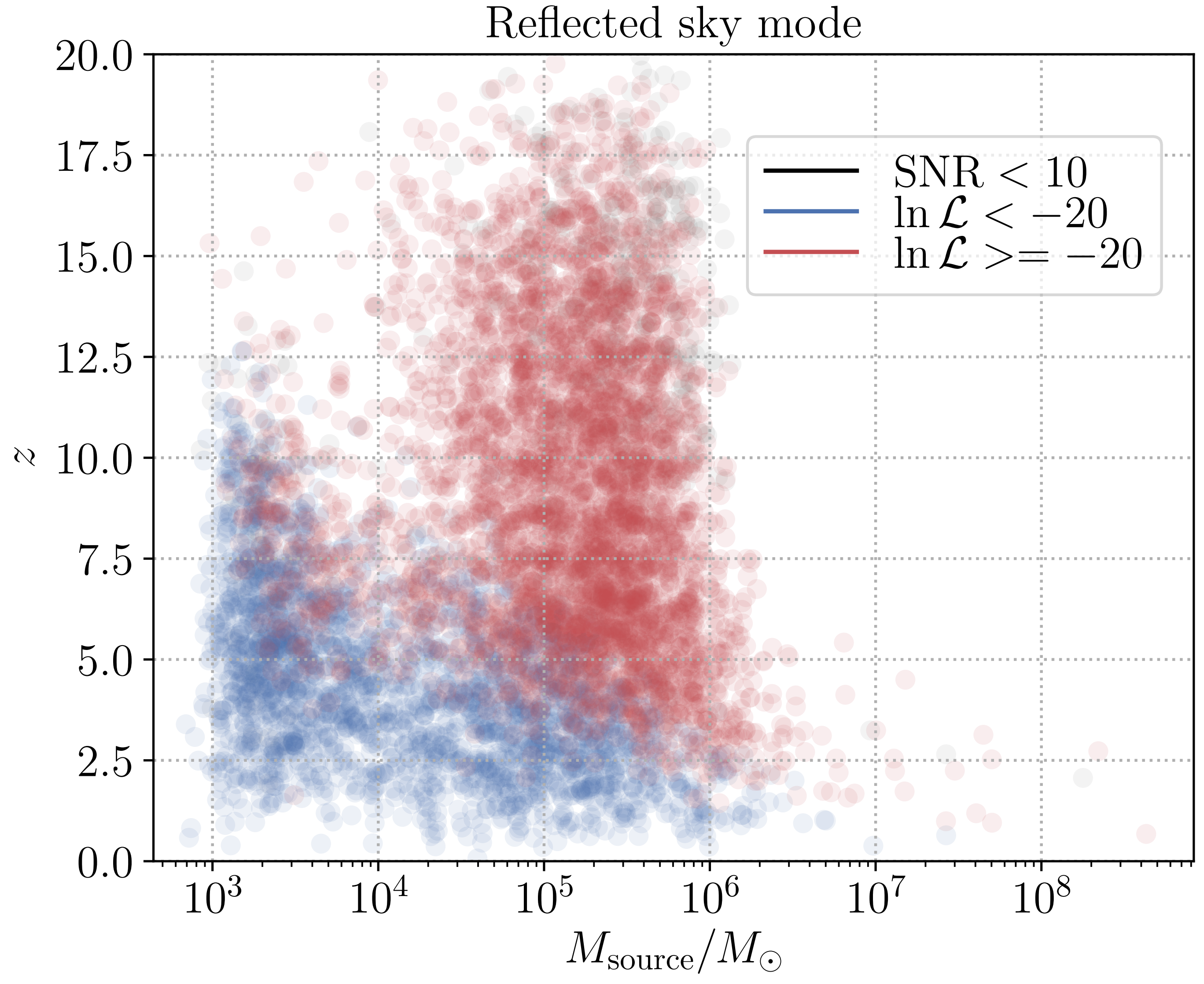
Approximate and conservative degeneracy measure: likelihood at the other sky positions

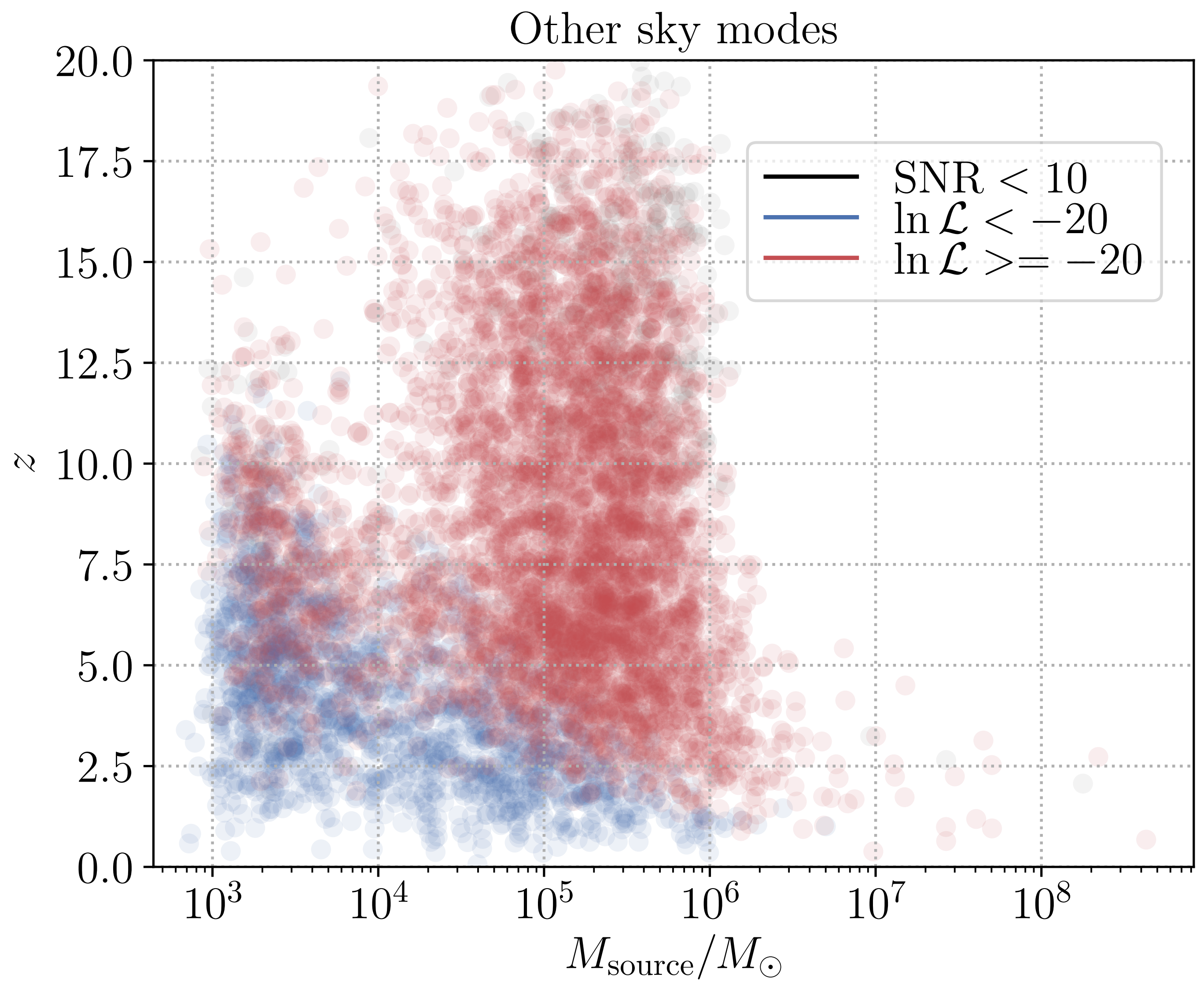
Threshold used here (a bit arbitrary):

$$\ln \mathcal{L} > -20$$

[Preliminary]

Pre-merger 5hr





Conclusion and outlook

Highlights

- Bayesian tools for fast PE of MBHB signals
- Explored the LISA parameter recovery of MBHBs
- Analyzed role of higher harmonics in breaking parameter space degeneracies
- Analyzed features of the instrumental response and breaking of multimodal sky degeneracies
- Successful analysis of MBHB in LISA Data Challenge 1 (Radler)

Outlook

- Explore the parameter space
- Optimize samplers for known degeneracies (MCMC jump proposals)
- More realistic analysis: multiple signals, realistic noise
- More realistic waveforms: precession and eccentricity
- Assess waveform requirements: how accurate do the waveforms need to be ?