Spectral separation of the stochastic background for LISA

GdR ondes gravitationnelles: groupe de travail "analyses de données"

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» Overview

Context: Stochastic background In LISA paper of BBH/BNS prediction

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- Stochastic Background : Superposition of a large number of independent sources (unresolved sources):
  - White dwarfs binaries in our galaxy
  - Binary Black Holes and Binary Neutron Stars from LIGO/Virgo Band
    - $\Omega_{GW} \simeq 1.8 \times 10^{-9} - 2.5 \times 10^{-9}$ at 25 Hz Chen *et al.* (2019)
    - $\Omega_{GW} \simeq 4.97 \times 10^{-9} - 2.58 \times 10^{-8}$ at 25 Hz Périgois *et al.* (2020)
  - Cosmological sources (early universe)
- Energy density spectrum:

Energy density spectrum

$$\Omega_{GW}(f) = \Omega_\alpha \left(\frac{f}{f_{ref}} \right)^\alpha$$

with $\alpha = 0$ for the cosmological component and $\alpha = 2/3$ for the astrophysical component.

Goal: Detecting a cosmological SGWB with LISA in the presence of an astrophysical background

MCMC: Markov chains Monte Carlo

» MCMC (Markov chain Monte Carlo)

* The likelihood function $p(d|\theta)$, (d = data, θ = parameter)

likelihood

$$\mathcal{L}(\mathbf{d}|\theta) = -\frac{1}{2}\sum_{k=0}^{N}\left[\frac{d_A^2}{S_A+N_A} + \frac{d_E^2}{S_E+N_E} + \frac{d_T^2}{N_T} + \ln\left(8\pi^3(S_A+NA)(S_E+N_E)N_T\right)\right]$$

* $p(\theta|d) \propto p(\theta)L(d|\theta)$
* Estimation of two components = fit of the parameters $\Omega_{Cosmo}, \Omega_{Galac}$ and $\alpha_{Cosmo}, \alpha_{Galac}$ and LISA noise N_{Pos} and N_{Acc}.

$\Longrightarrow$ using a Metropolis-Hasting sampler

Fisher Information Matrix

» Fisher Information Matrix

Fisher Information Matrix

$$F_{ab} = \frac{1}{2}\mathrm{Tr}\left(\mathcal{C}^{-1}\frac{\partial\mathcal{C}}{\partial\theta_a}\mathcal{C}^{-1}\frac{\partial\mathcal{C}}{\partial\theta_b}\right)$$

$$= \frac{1}{2}\sum_{I=A,E,T}\sum_{k=0}^{N}\frac{\frac{\partial S_I(f)+N_I(f)}{\partial\theta_a}\frac{\partial S_I(f)+N_I(f)}{\partial\theta_b}}{\left(S_I(f)+N_I(f)\right)^2}$$

Co-variance Matrix

$$\mathcal{C}(\theta, f) = \begin{pmatrix} S_A+N_A & 0 & 0 \\ 0 & S_E+N_E & 0 \\ 0 & 0 & N_T \end{pmatrix}$$

with N_I is the LISA noise of the channel $I = [A, E, T]$ and $S_I \propto \sum_\alpha \Omega_\alpha \left(\frac{f}{f_{ref}}\right)^\alpha$ the SGWB.

$\Longrightarrow \sqrt{F^{-1}_{aa}} = \sigma_a$

Results 6 parameter A-MCMC

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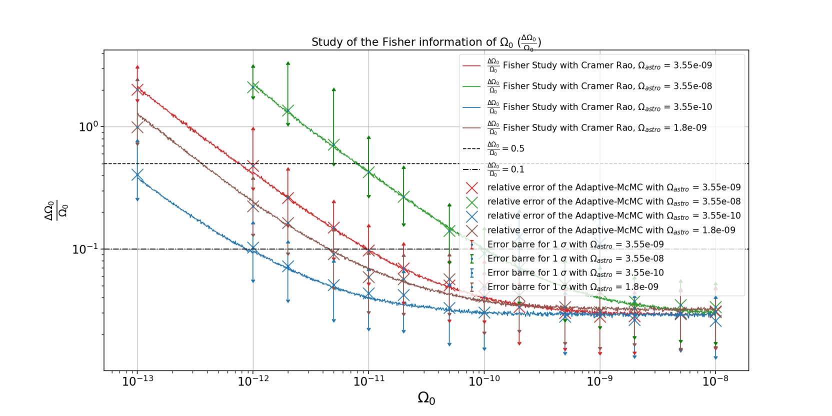


Uncertainty of the estimation of the Cosmological Amplitude from the Fisher Information study in line (with the Cramer-Rao calculation) and the parametric estimation from the A-MCMC in scatters for the channel A with the noise channel T. The upper horizontal dash line represent the error level 50%. In fact, above the line, the error is greater than 50%.

» Results 6 parameter A-MCMC

Prediction of the measurement limit of Cosmological Amplitude in 4 contexts of Isotropic astrophysical background In the LISA noise context (acceleration noise : $N_{Acc} = 9 \times 10^{-30}$ m^{2}s^{-4}Hz^{-1} and Optical Metrology System noise $N_{Opt} = 2.25 \times 10^{-22}$ m^{2}Hz^{-1}) of 4 years mission data measurement:

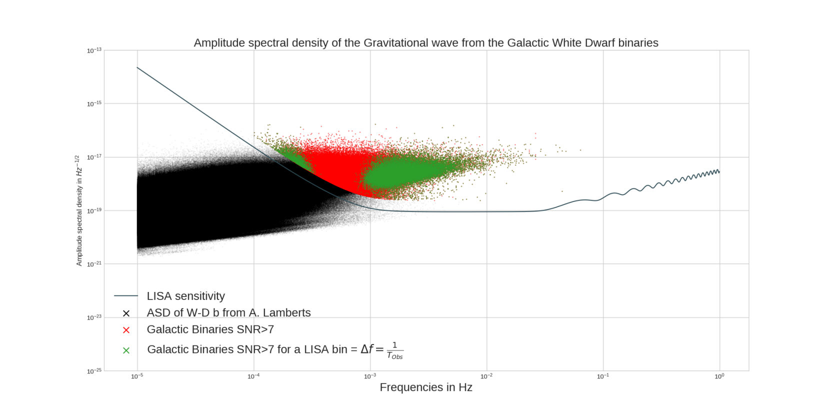
Limit for BBH/BNS + Cosmo + LISA noise

* $\Omega_{astro} = 3.55 \times 10^{-8}$ (25 Hz): $\Omega_{Cosmo,lim} = 7.8 \times 10^{-12}$
* $\Omega_{astro} = 3.55 \times 10^{-9}$ (25 Hz): $\Omega_{Cosmo,lim} = 7.8 \times 10^{-13}$
* $\Omega_{astro} = 1.8 \times 10^{-9}$ (25 Hz): $\Omega_{Cosmo,lim} = 3.6 \times 10^{-13}$
* $\Omega_{astro} = 3.55 \times 10^{-10}$ (25 Hz): $\Omega_{Cosmo,lim} = 7.6 \times 10^{-14}$

$\Longrightarrow$ paper in preprint (arXiv:2011.05055).

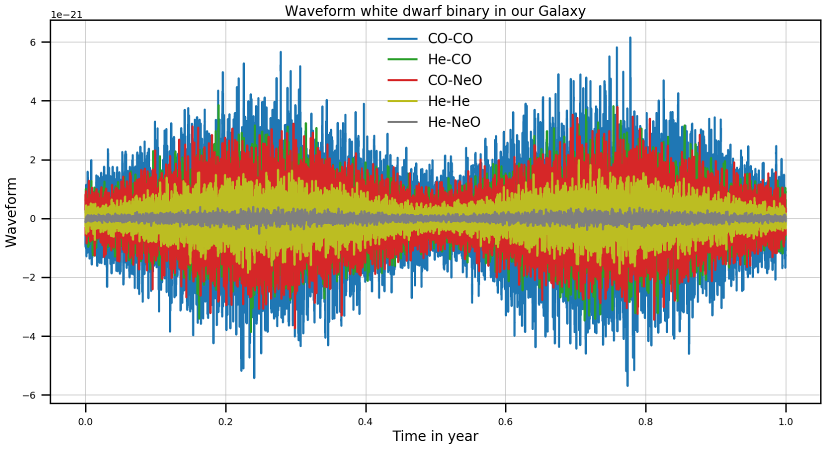
Orbital Modulation of the White dwarf binaries in our Galaxy

» Population of the White dwarf binaries in our Galaxy



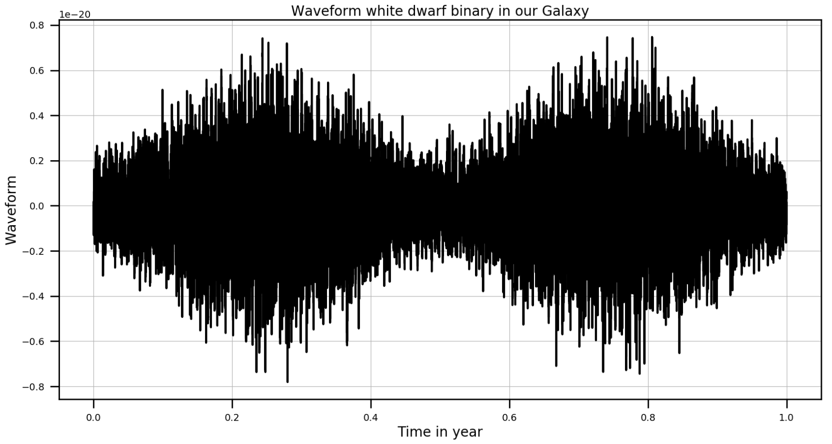
ASD of the White dwarf binaries in our Galaxy from A. Lamberts (2019), **black**: $\simeq$ 35 000 000 binaries, **red**: binaries $SNR > 7$ and **green**: $SNR > 7$ + LISA bin ($\simeq$ 32 000 binaries)

» Orbital Modulation of the White dwarf binaries in our Galaxy



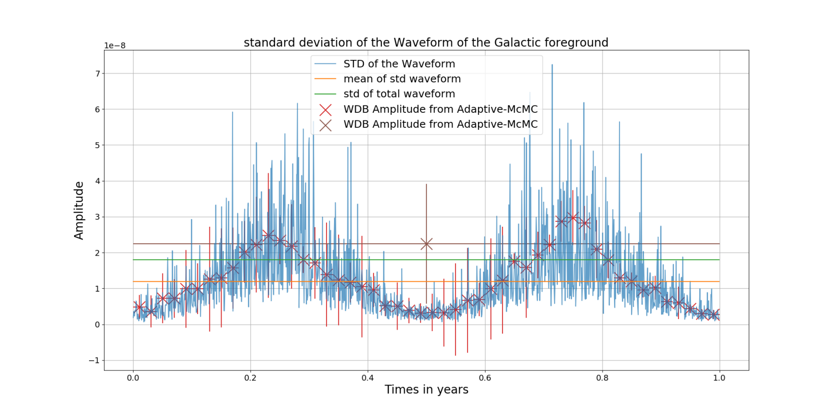
Gravitational signal of the white-Dwarf binaries according to the types of population seen by LISA

» Orbital Modulation of the White dwarf binaries in our Galaxy



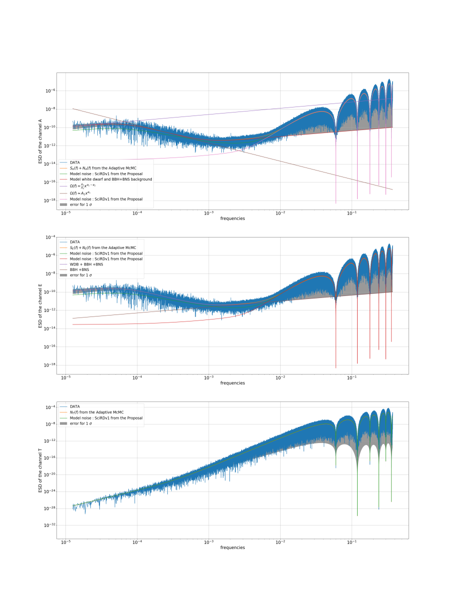
Total Gravitational signal of the white-Dwarf binaries seen by LISA

» Orbital Modulation of the White dwarf binaries in our Galaxy



Measurement of the orbital Modulation of the White dwarf binaries amplitude. In **blue**: $\Omega_A = \frac{4\pi^2}{3H_0^2}\left(\frac{c}{2\pi L}\right)^2 STD(h(t))$. In **red** scatter : 50 A-MCMC of 8 parameters (BBH + WD + LISA noise).

» A-MCMC of 8 parameters (BBH + WD + LISA noise)



Input of the MCMC:

* $A_3 = 4.4 \times 10^{-12}$ ($f_* = \frac{2\pi L}{c} \simeq 0.019$ Hz) , $\alpha_3 = 2/3$
* data model:
$$S(f) = \frac{A_1 \left(\frac{f}{f_*}\right)^{\alpha_1}}{1 + A_2 \left(\frac{f}{f_*}\right)^{\alpha_2}} + A_3 \left(\frac{f}{f_*}\right)^{\alpha 3}$$

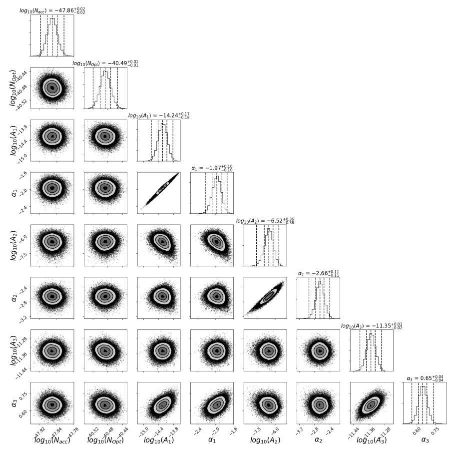
For $1 \ll A_2 \left(\frac{f}{f_*}\right)^{\alpha_2}$ (low frequency):

Energy spectral density of White dwarf binaries at low frequency

$$\Omega_{WD}(f) \simeq \frac{A_1}{A_2} \left(\frac{f}{f_*}\right)^{\alpha_1 - \alpha_2}$$

At low frequency, it can be approximated by a power law function, for a white dwarf binaries foreground the slope $\alpha = \alpha_1 - \alpha_2 = \frac{2}{3}$.

» A-MCMC of 8 parameters (BBH + WD + LISA noise)



Ouput of the A-MCMC

* BBH/BNS:
  * $\Omega = 4.46 \times 10^{-12} \pm 1.2 \times 10^{-13}$
  * $\alpha = 0.65 \pm 0.04$
* WD low frequencies:
  * $\Omega = 1.94 \times 10^{-8} \pm 1.33 \times 10^{-8}$
  * $\alpha = 0.68 \pm 0.06$

10 parameters runs of A-MCMC (LISA noise + BBH/BNS + DWD + Cosmo)

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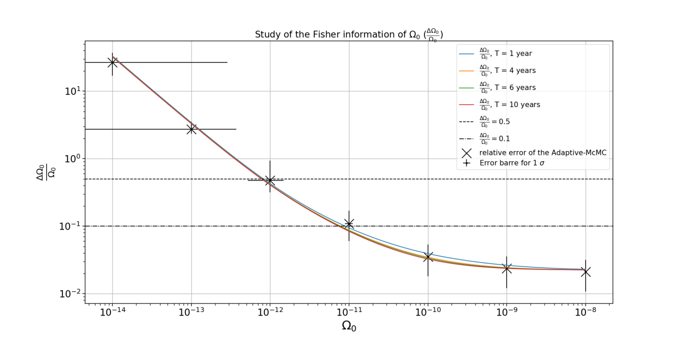

Uncertainty Estimation of the Cosmological Amplitude from the Fisher Information study in line (for 4 times duration) and the parametric estimation from the A-MCMC in scatters for the channel A with the noise channel T. The upper horizontal dash line represent the error level 50%. In fact, above the line, the error is greater than 50%.

» 10 parameters runs of A-MCMC (LISA noise + BBH/BNS + WD + Cosmo)

Times effect

We see no significant influence of the time duration (4,6 and 10 years) on the Cosmological measurement

Measurement limit

$\Omega_{Cosmo,lim} = 8 \times 10^{-13}$

Limitation

BBH and BNS principal limitation for the Cosmological background

$\Longrightarrow$ paper in preparation

Conclusion

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We provide evidence that it is possible to measure the cosmological SGWB :

Measurement limit

$\Omega_{Cosmo,lim} = 8 \times 10^{-14} - 8 \times 10^{-12}$

Limitation

BBH and BNS principal limitation for the Cosmological background

The End : Thank You !