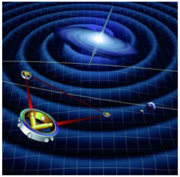
DE LA RECHERCHE À L'INDUSTRIE

CEA - Saclay

Sparse data inpainting for gapped data in LISA



Aurore BLELLY

November 30, 2020

www.cea.fr

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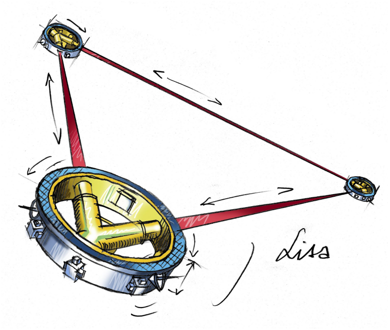
Laser Interferometer Space Antenna

A Space Interferometer (ESA Mission)

- 3 satellites 2.5 millions km one from another
- Follows the movement of the Earth on its orbit
- Probes a **frequency range that is still unexplored** ⇒ high potential for **new discoveries**
- Launching: 2034



[1]

[1] https://www.esa.int/Science_Exploration/Space_Science/LISA

Data Characteristics

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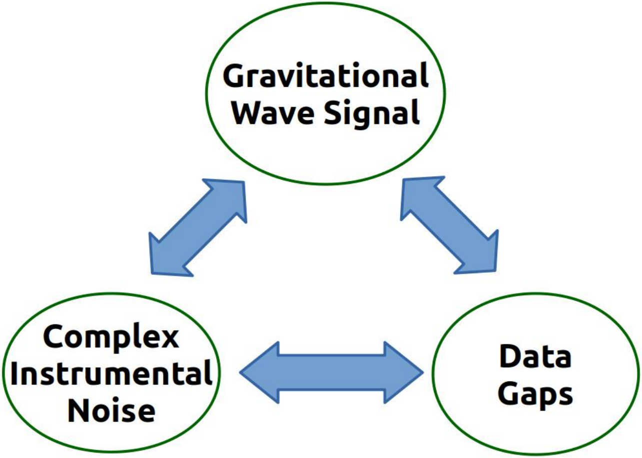

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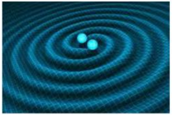
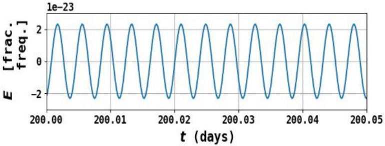
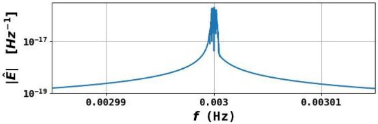
Gravitational Wave Signal





Focus on GBs :

- 10^6 expected sources
- Goal : identify ~ 20.000
- Low SNR = long observation required
- **Neary mono-frequency signal**



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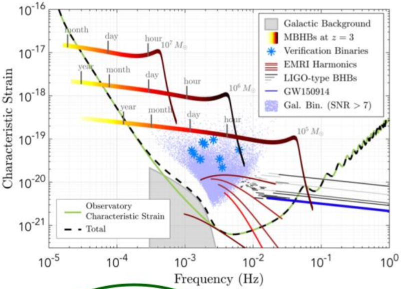
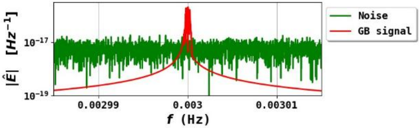


Noise caracteristics :

- **Known expected distribution**
- For now, simplified hypothesis (Gaussianity)
- Noise level impacts the ability to detect a source

Complex Instrumental Noise



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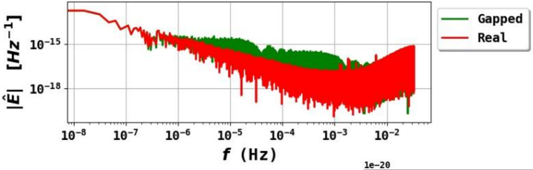

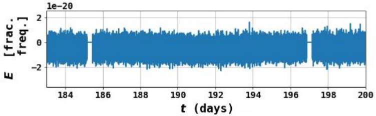

Data gaps :

- Maintenance or unplanned
- Various durations/frequencies :
  - Planned : 7 hours / 2 weeks
  - Unplanned : a few seconds => several days
- **Impact expected noise distribution**
- **Impact** quality of identification

Data Gaps

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- **Goal:** Mitigating the impact of gaps in LISA data by filling the gaps with as little new information as possible.
- **State of the art:** Data augmentation method [2] : based on sampling and MCMC.
- **Strategy:** Using the **waveform expected structure** instead of **parametric description**.

⇒ **Non-parametric** method.

[2] Q. Baghi, I. Thorpe, J. Slutsky, J. Baker, T. Dal Canton, N. Korsakova, N. Karnesis, *Gravitational-wave parameter estimation with gaps in LISA: a Bayesian data augmentation method*, 2019

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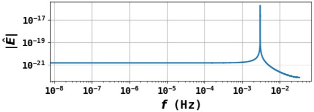
Prior hypothesis:

- Simple representation in Fourier basis (**Sparsity prior**)



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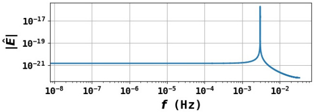
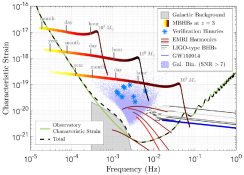


- Noise is Gaussian with known PSD Σ **for ungapped data**



Work Hypothesis

Prior hypothesis:

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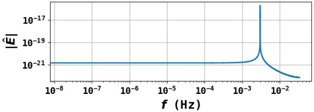

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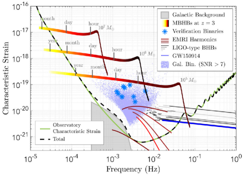

- Gap time is known:

Sparse Data Inpainting

Sparse Data Inpainting Method

Cost function with data gaps

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- M is the data mask: 

- (S, N) the joint estimate of **ungapped** signal and noise

$$(S, N) =$$

Sparse Data Inpainting Method

Cost function with data gaps

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- M is the data mask: 

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$$(S, N) = \underbrace{\underset{Y=M(S+N)}{\text{Argmin}}}_{\text{Data Adequacy}} \Big[$$

Sparse Data Inpainting Method

Cost function with data gaps

- M is the data mask: 

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$$(S, N) = \underbrace{\underset{Y=M(S+N)}{\text{Argmin}}}_{\text{Data Adequacy}} \left[\underbrace{\left\|\gamma \odot \Sigma^{-1/2}\widehat{S}\right\|_{12}}_{\text{Solution prior Sparsity}} + \right.$$

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With Y the gapped measurement; Σ the noise PSD and γ the threshold.

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With Y the gapped measurement; Σ the noise PSD and γ the threshold.

- **Resolution:** alternates between signal estimation and noise estimation.

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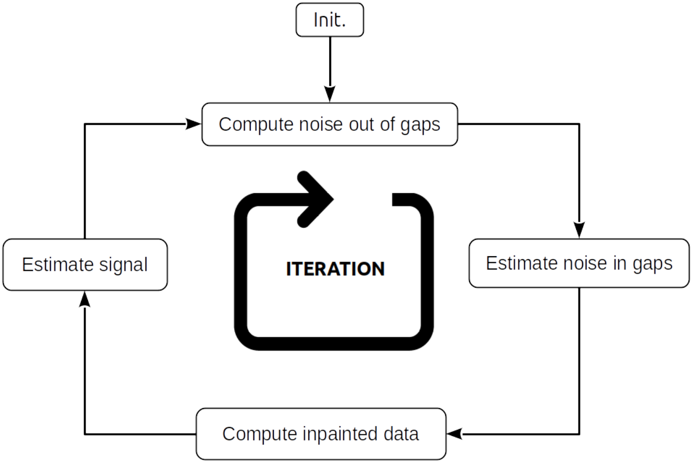

Algorithm Structure
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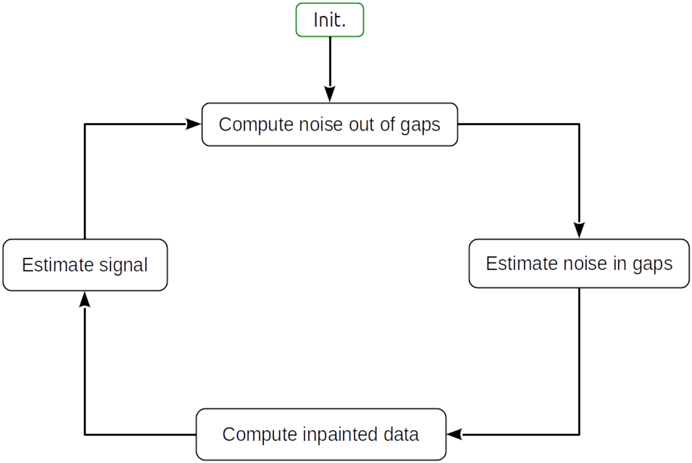

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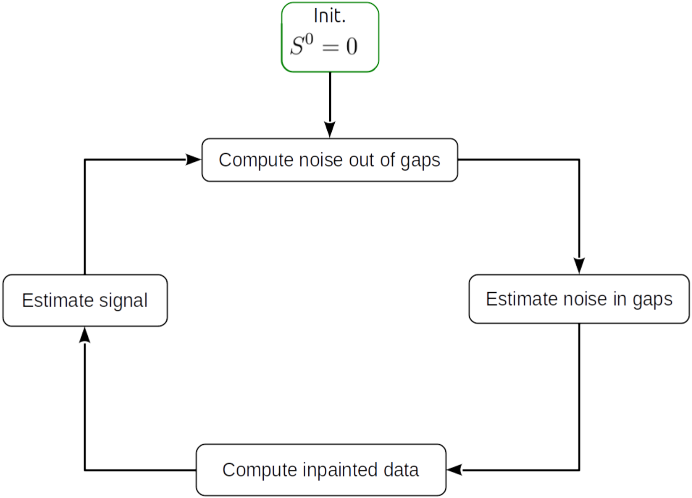

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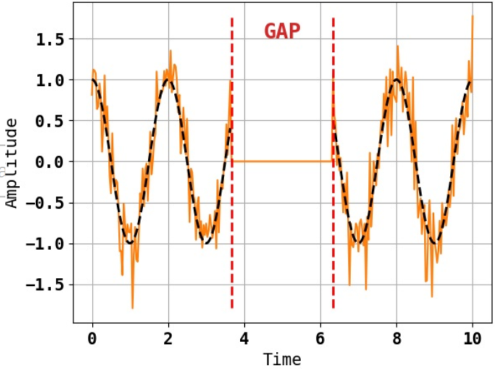
$S^0 = 0$

Estima



n gaps

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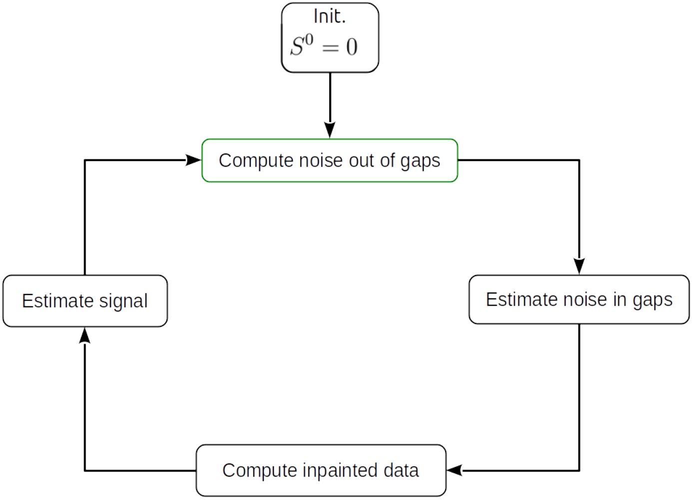

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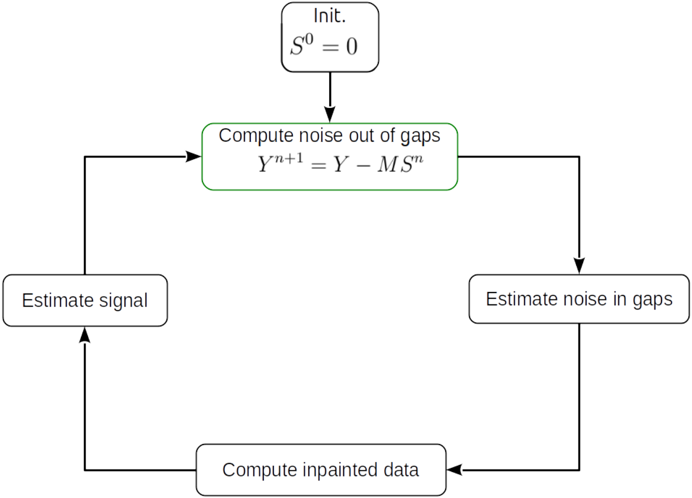

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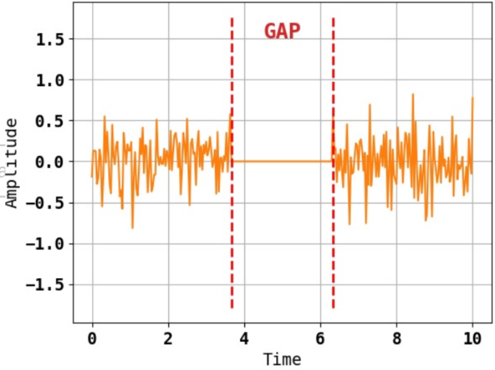

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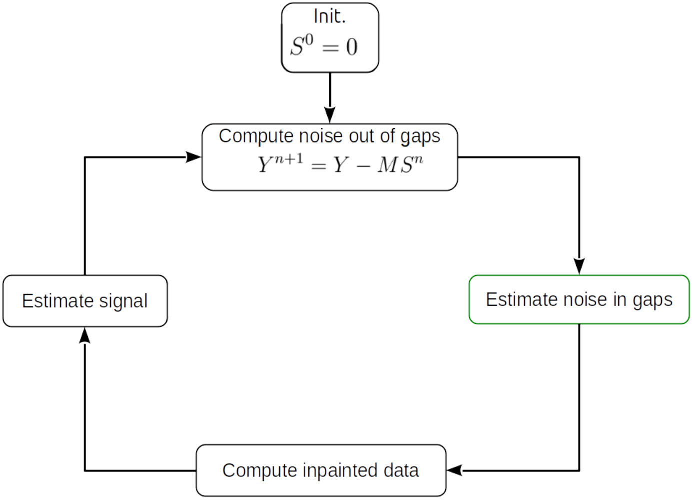

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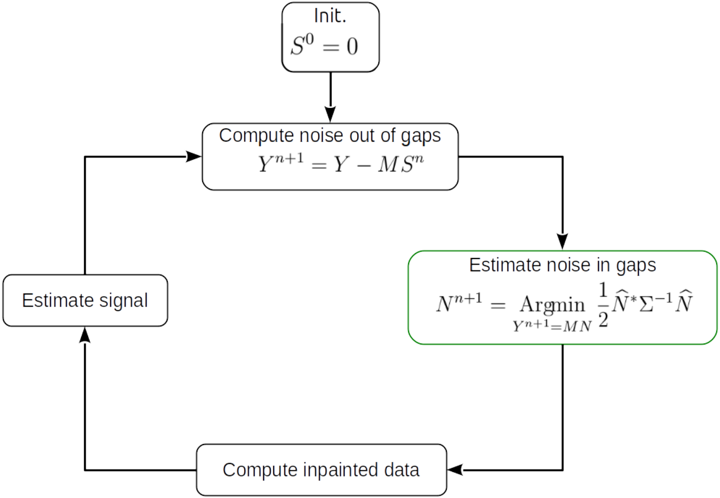

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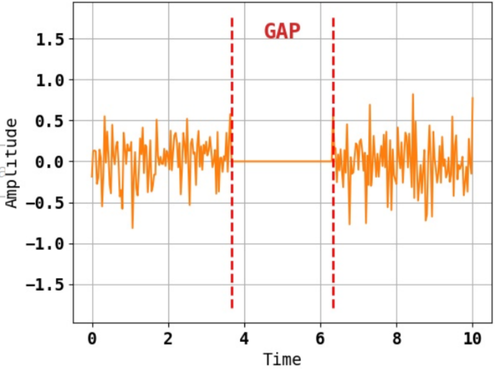

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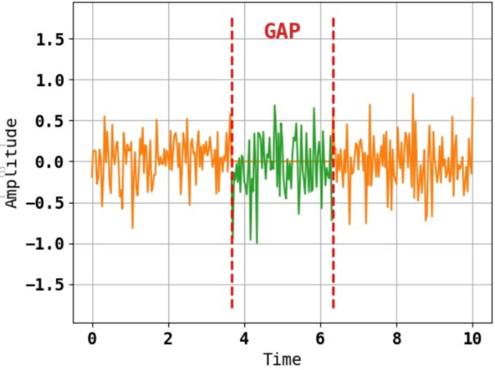

gaps
$\widehat{V}^*\Sigma^{-1}\widehat{N}$

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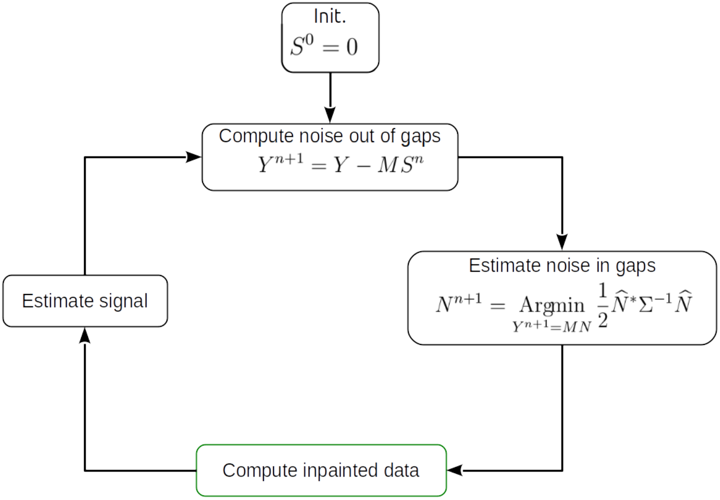

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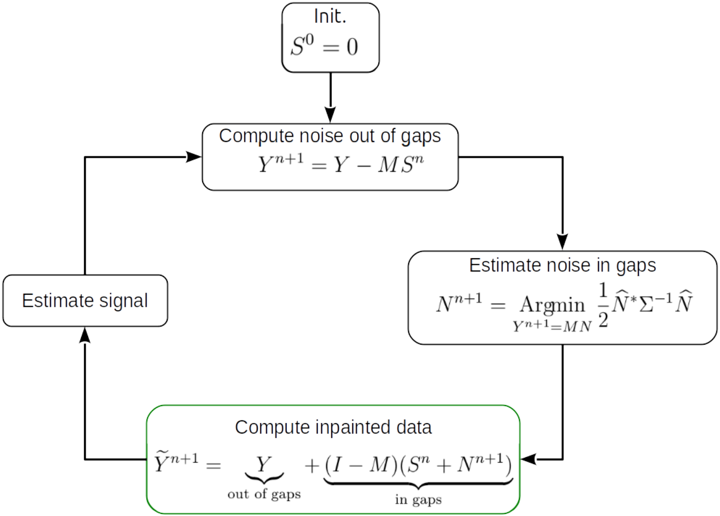

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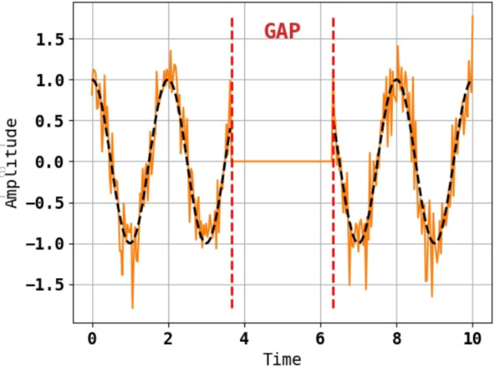

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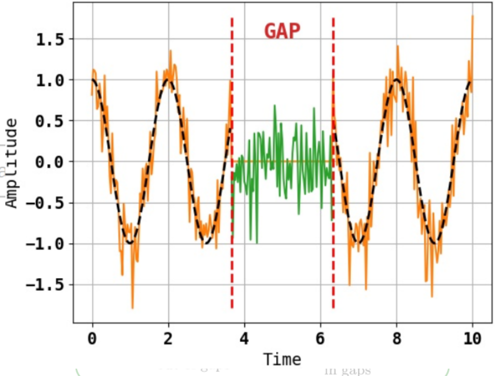

in gaps

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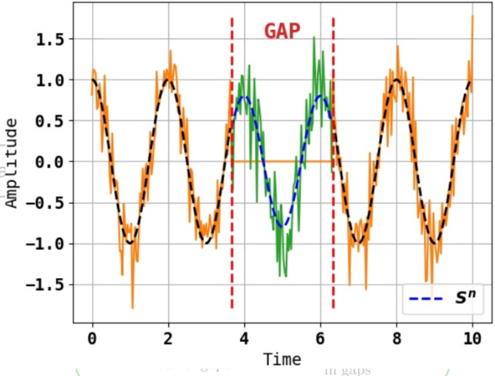

in gaps

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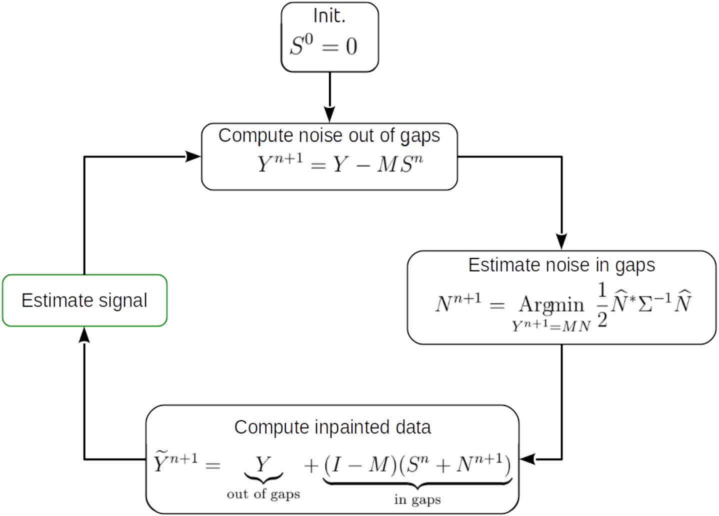

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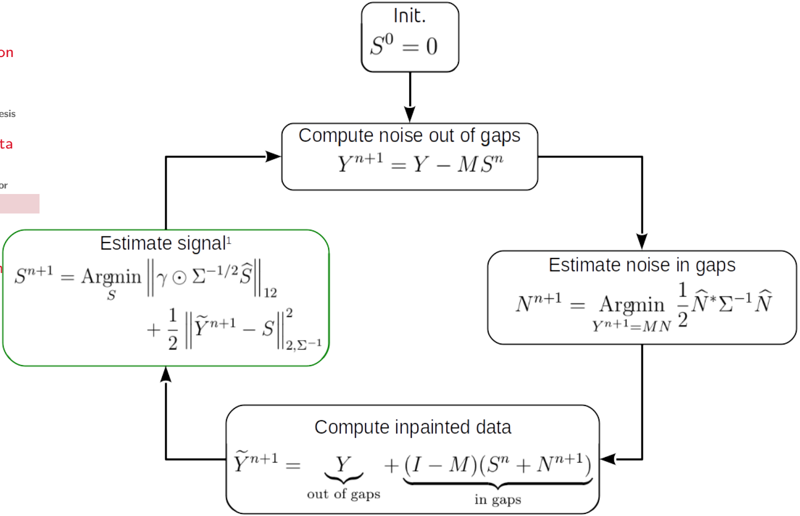

[1] A. Blelly, J. Bobin and H. Moutarde, *Sparsity Based Recovery of Galactic Binaries Gravitational Waves*, 2020
https://github.com/GW-IRFU/gw-irfu/

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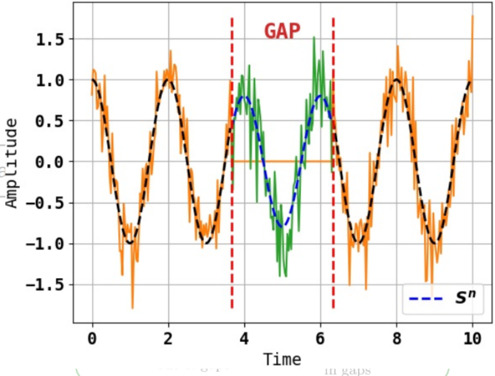

in gaps

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Init.
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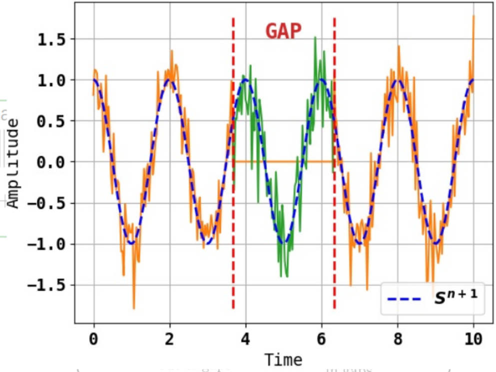
Estima
$S^{n+1} = \underset{S}{\text{Argmin}} \|$

gaps
$\widehat{V}^*\Sigma^{-1}\widehat{N}$

in gaps



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Init.

$$S^0 = 0$$

Compute noise out of gaps

$$Y^{n+1} = Y - MS^n$$

Estimate noise in gaps

$$N^{n+1} = \underset{Y^{n+1}=MN}{\text{Argmin}} \ \frac{1}{2}\widehat{N}^* \Sigma^{-1} \widehat{N}$$

Compute inpainted data

$$\widetilde{Y}^{n+1} = \underbrace{Y}_{\text{out of gaps}} + \underbrace{(I-M)(S^n + N^{n+1})}_{\text{in gaps}}$$

Estimate signal[1]

$$S^{n+1} = \underset{S}{\text{Argmin}} \left\| \gamma \odot \Sigma^{-1/2} \widehat{S} \right\|_{12} + \frac{1}{2} \left\| \widetilde{Y}^{n+1} - S \right\|^2_{2,\Sigma^{-1}}$$

[1] A. Blelly, J. Bobin and H. Moutarde, *Sparsity Based Recovery of Galactic Binaries Gravitational Waves*, 2020
https://github.com/GW-IRFU/gw-irfu/

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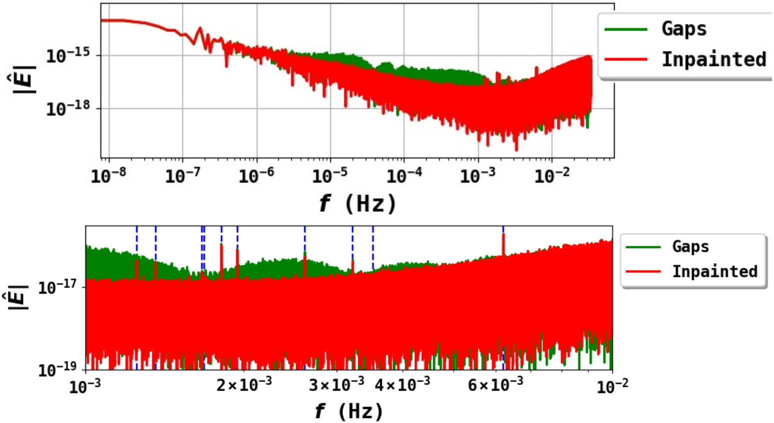

- **LDC 1-3:** 10 Verification galactic binaries
- Gaps: 7h/2 weeks (planned), 10 min / 1 day (unplanned)

[3] https://lisa-ldc.lal.in2p3.fr/

Sparse Data Inpainting Method

Impact on input signal spectrum - LDC1-3 [3]

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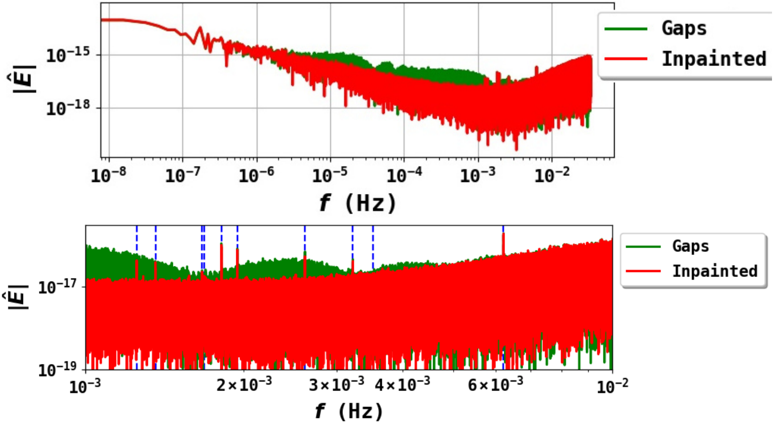

$\Rightarrow$ Effective noise PSD is flattened

$\Rightarrow$ **Limited noise leakage** thanks to inpainting process

[3] https://lisa-ldc.lal.in2p3.fr/

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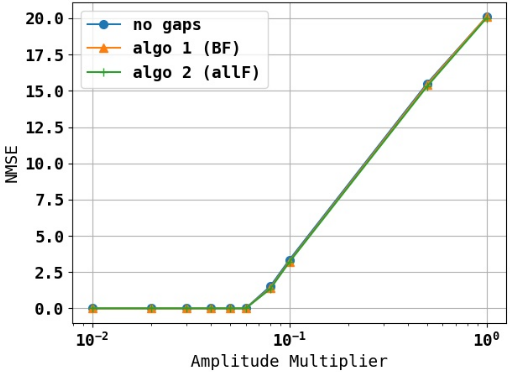

- Single GB, solution for different amplitudes
- Extracted signal from gapped data has the same quality as the one extracted from ungapped signal

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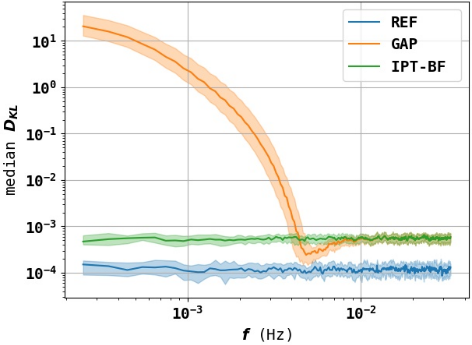

Figure: 7 hours / 2 weeks gaps: Kullback-Leibler Divergence

The Kullback-Leibler Divergence measures a discrepancy between the expected and the recovered noise distributions.

Kullback-Leibler divergence

Recovered noise follows the expected distribution

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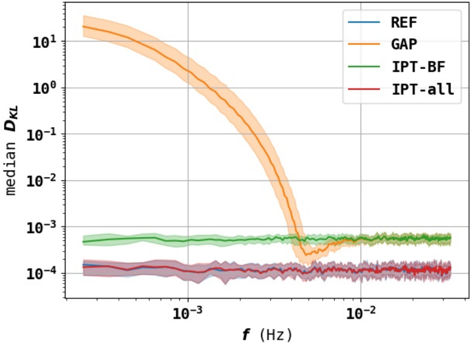

Figure: 7 hours / 2 weeks gaps: Kullback-Leibler Divergence

The Kullback-Leibler Divergence measures a discrepancy between the expected and the recovered noise distributions.

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- Tests with both **gapped** and **ungapped** data.
- **Efficient reconstruction** of the signal in the gaps
- Fast: ~ 1 hour for 2 years of data on a laptop
- Works as well with a **large unknown number of separated sources**.
- **Flexible framework** which could be adapted to:
  - Estimation of power spectral density.
  - Other types of sources.
  - Sources of different types.