Beam Dynamics in MBA Lattices with Different Chromaticity Correction Schemes

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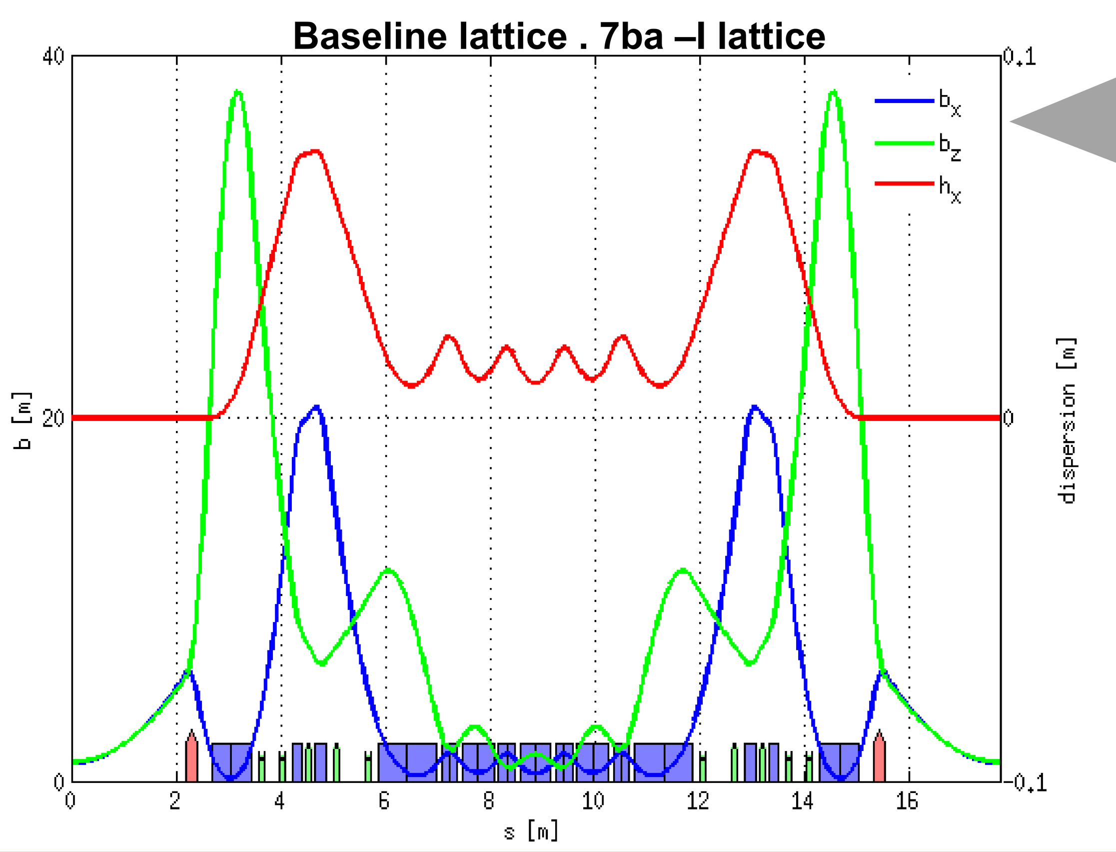
Abstract

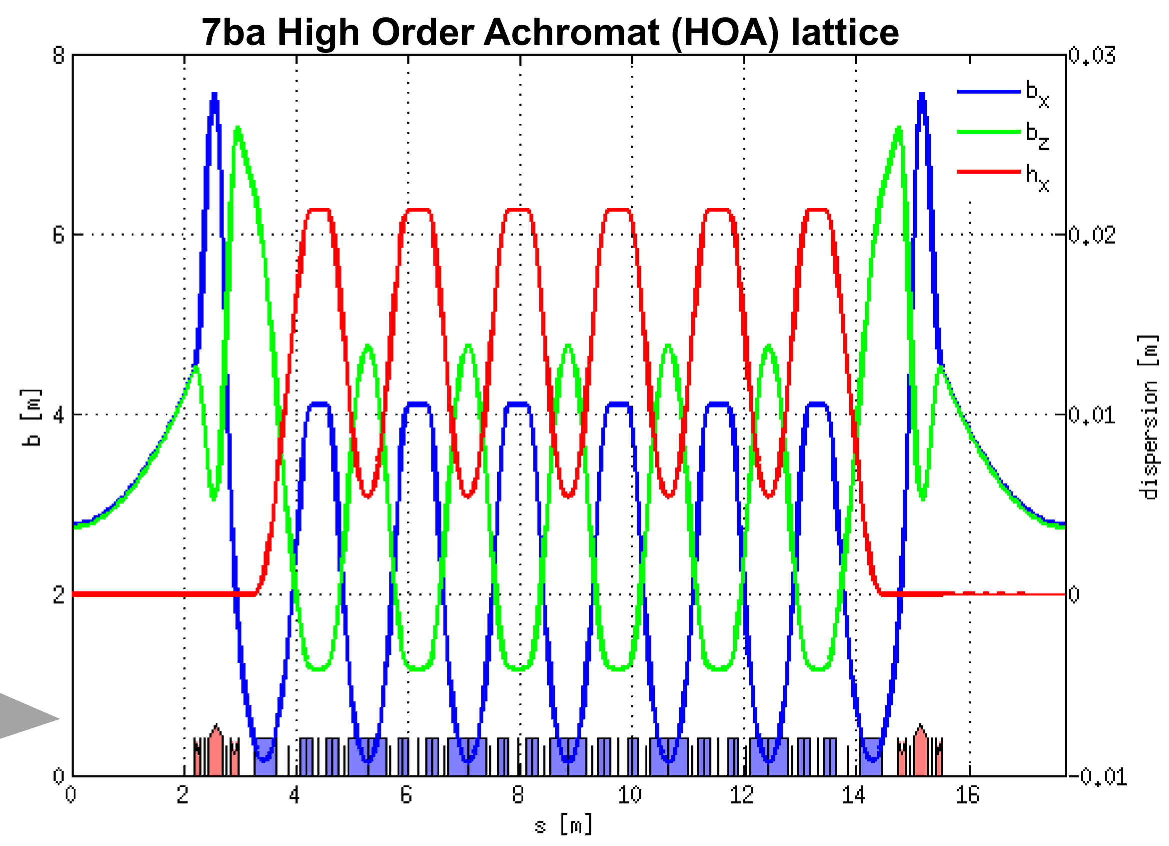
Ultra-low emittance lattices are being studied for the future upgrade of the SOLEIL 2.75 GeV storage ring. The candidate baseline lattice is inspired by the ESRF-EBS-type MBA lattice, introducing a (–I) transformation to compensate the nonlinear impact of sextupoles thanks to the lattice symmetry and tight control of the betatron phase advance between sextupoles. Whilst its performance is under study, other types of lattices are being developed for SOLEIL: in particular, the so-called High-Order Achromat (HOA) lattice. Though the (–I) scheme provides a large on-momentum transverse dynamic aperture in 4D, its off-momentum performance is limited. Further studies in 6D reveal intrinsic off-momentum transverse oscillations, which are considered to result from of a nonlinear increase of the path length. The effect of the inhomogeneous sextupole distribution in the (–I) scheme shall be presented and compared with the HOA lattice under study.



The baseline lattice uses the \$-I\$ transformation, setting a $((2k+1)\pi, n\pi)$ phase advance between two sextupoles (with $k, n \in \mathbb{N}$) for:

- kick cancellation
- optimised on-momentum acceptance
+ dispersion bumps at the location of the sextupoles for:
- increased efficiency
- global chromaticity correction.

The so-called HOA lattice is built in several identical small cells, where phase advance is fixed in both planes to cancel geometric resonances over each cell. ensure the cancellation of resonances up to the third order. Each cell owns its pair of sextupoles for local chromaticity correction.



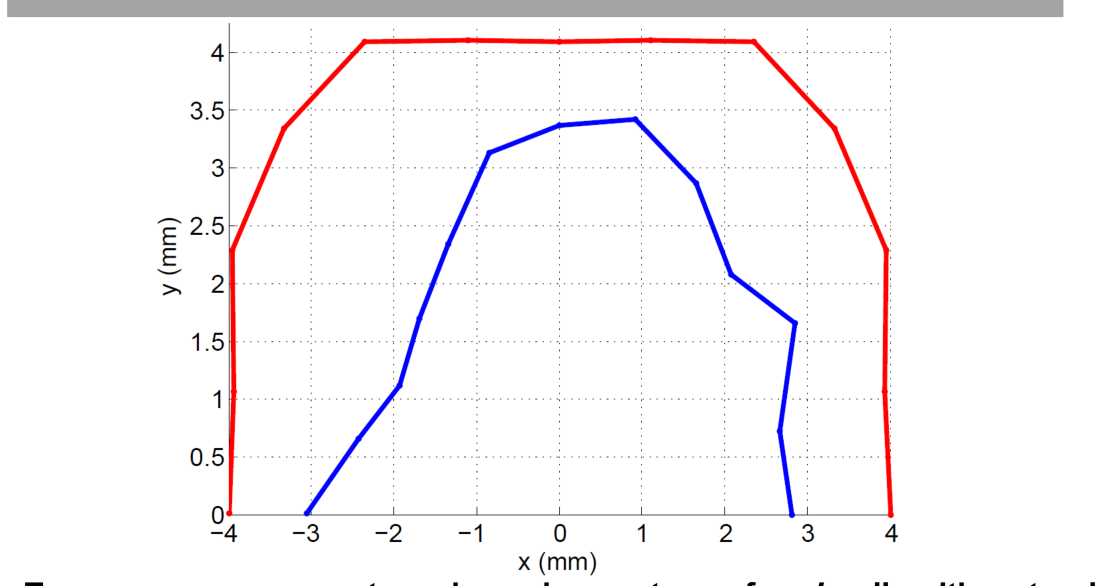
PATH LENGTHENING EFFECT AND FIRST-ORDER PERTURBATION THEORY

PATH LENGTHENING EFFECT

A strong coupling between the longitudinal and the transverse planes in the –I lattice makes a particle go off-energy each turn - by increased path length, and falls out of the off-momentum acceptance, reducing the transverse dynamic aperture (see below).

The path lengthening due to large amplitude betatron motions is studied in both lattices. The concerned effect depends on the chromaticity (ξ_x, ξ_y) in both planes :

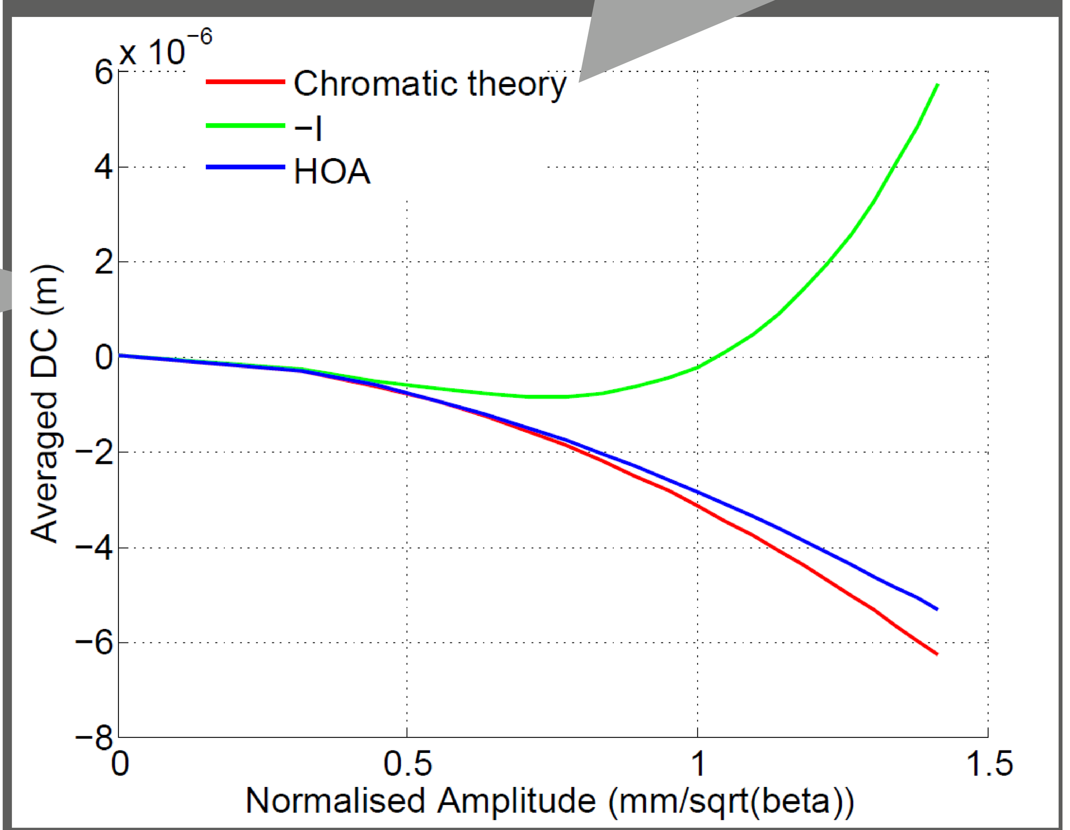
$$\Delta C = -2\pi(J_x\xi_x + J_y\xi_y)$$



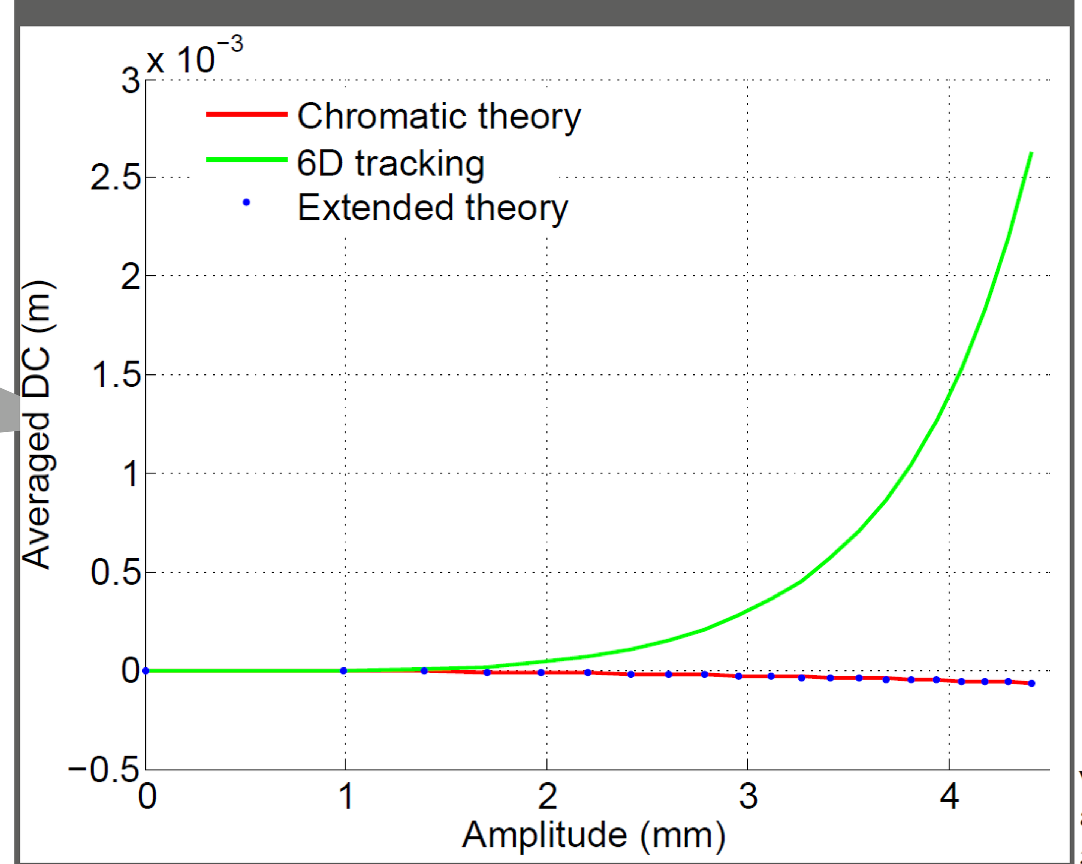
Transverse on-momentum dynamic aperture of a -I cell, with natural chromaticities. In red, the lattice without RF, in blue, RF system added, with a voltage of 1.1 MV.

Higher-order terms will be required to describe the tracked path length of the low emittance lattices under study, as the first-order theory remains close to the general chromatic theory. The second order in perturbation, neglected in this contribution for the sake of simplicity, appears to be responsible for the path length effect in the low emittance lattices.

The path length of the –I lattice (green) follows a rule different from the known chromatic dependence (red). The effect is less prominent in the HOA lattice (blue).



Path lengthening as a function of amplitude after 1 turn, averaged on the input phase, for both –I and HOA lattices, at a corrected chromaticity of (1,1).



Averaged path lengthening considering the three described methods -- linear formula of ΔC in red, 6D tracking of the -I lattice in green and perturbed path length using the distorted trajectory on the side, for corrected chromaticity \$(1,1)\$.

PERSPECTIVES

Higher-order theory, if obtained, will be used to reduce or even cancel the path lengthening effect observed in the -I lattice. This optimisation may be done, either analytically or numerically using sextupoles, under the constraint of correcting the chromaticity, in order to restore its on-momentum performance.

FIRST-ORDER PERTURBATION THEORY

Using the first-order perturbation theory described by M. Takao in [3], the distortion of the averaged trajectory coordinates (x, x') can be derived according to dipolar and sextupolar gradients. The perturbed Hamiltonian considered is :

$$H = \frac{p_x^2 + p_y^2}{2} + \frac{1}{2}(k_x^2 + g_0)x^2 - \frac{1}{2}g_0 y^2 + \frac{g_1}{3!}(x^3 - 3xy^2) + \frac{1}{2}k_x x(p_x^2 + p_y^2),$$

Distorted averaged trajectory coordinates are :

$$\begin{aligned}\langle x(s)\rangle_{\phi_x} = &-\frac{J_x\sqrt{\beta_x}}{4\sin(\pi\nu_x)}\int_s^{s+C} ds'\sqrt{\beta_x}\,(g_1\beta_x + k_x\gamma_x)\cos\left(\overline{\psi}(s',s)\right)\\ &-\frac{J_y\sqrt{\beta_x}}{4\sin(\pi\nu_x)}\int_s^{s+C} ds'\sqrt{\beta_x}\,(-g_1\beta_y + k_x\gamma_y)\cos\left(\overline{\psi}(s',s)\right)\\ &-\frac{J_x\sqrt{\beta_x}}{2\sin(\pi\nu_x)}\int_s^{s+C} ds' k_x\beta_x^{-1/2}\alpha_x\left(\sin\left(\overline{\psi}(s',s)\right) + \alpha_x\cos\left(\overline{\psi}(s',s)\right)\right)\\ &+\frac{J_x\sqrt{\beta_x}}{4\sin(\pi\nu_x)}\int_s^{s+C} ds' k_x\beta_x^{-1/2}\cos\left(\overline{\psi}(s',s)\right)\end{aligned}$$

and

$$\langle x'^2(s)\rangle_{\phi_x} = J_x\gamma_x + k_x^2\alpha_x^2J_x^2 + J_x^2k_x^2\frac{\beta_x\gamma_x}{2} + 2k_x\alpha_x\sqrt{\frac{2J_x}{\beta_x}}\left(1 + \frac{\sqrt{J_x}}{2}\right)P_1 + k_x\sqrt{\frac{2J_x}{\beta_x}}\left(1 + \frac{\sqrt{J_x}}{2}\right)P_2,$$

with

$$\begin{aligned}P_1 = &\frac{J_x^{3/2}}{4\sqrt{2}\sin(\pi\nu_x)}\int_s^{s+C} ds'\sqrt{\beta_x}\\ &\times\left(g_1\beta_x + \ldots \beta_x\right)\\ &\times\left(\sin\left(\overline{\psi}(s',s)\right) - \alpha_x(s)\cos\left(\overline{\psi}(s',s)\right)\right)\\ &-\frac{J_x^{3/2}}{2\sqrt{2}\sin(\pi\nu_x)}\int_s^{s+C} ds' k_x\sqrt{\beta_x}\gamma_x\alpha_x\\ &\times(\alpha_x\alpha_x(s) + 1)\cos\left(\overline{\psi_x}(s',s)\right)\\ &-\frac{J_x^{3/2}}{2\sqrt{2}\sin(\pi\nu_x)}\int_s^{s+C} ds' k_x\sqrt{\beta_x}\gamma_x\alpha_x\\ &\times(\alpha_x - \alpha_x(s))\sin\left(\overline{\psi}(s',s)\right)\\ &+\frac{J_x^{1/2}J_y}{4\sqrt{2}\sin(\pi\nu_x)}\int_s^{s+C} ds'\sqrt{\beta_x}\,(-g_1\beta_y + k_x\gamma_y)\\ &\times\left(\sin\left(\overline{\psi}(s',s)\right) - \alpha_x(s)\cos\left(\overline{\psi}(s',s)\right)\right),\end{aligned}$$

and

$$\begin{aligned}P_2 = &-\frac{J_x^{3/2}}{4\sqrt{2}\sin(\pi\nu_x)}\int_s^{s+C} ds'\sqrt{\beta_x}\beta_x(s)\gamma_x(s)\\ &\ldots + k_x\left(\gamma_x - \frac{1}{\beta_x}\right)\Big)\cos\left(\overline{\psi}(s',s)\right)\\ &-\frac{J_x^{1/2}J_y}{4\sqrt{2}\sin(\pi\nu_x)}\int_s^{s+C} ds'\sqrt{\beta_x}\beta_x(s)\gamma_x(s)\\ &\times(-g_1\beta_y + k_x\gamma_y)\cos\left(\overline{\psi}(s',s)\right)\\ &-\frac{J_x^{3/2}}{2\sqrt{2}\sin(\pi\nu_x)}\int_s^{s+C} ds' k_x\beta_x^{-1/2}\alpha_x\beta_x(s)\gamma_x(s)\\ &\times\left(\sin\left(\overline{\psi_x}(s',s)\right) + \alpha_x\cos\left(\overline{\psi_x}(s',s)\right)\right)\\ &-\frac{3J_x^{3/2}}{2\sqrt{2}\sin(3\pi\nu_x)}\int_s^{s+C} ds' k_x\beta_x^{-1/2}\alpha_x\beta_x(s)\gamma_x(s)\\ &\times\left(\sin\left(3\overline{\psi}(s',s)\right) - \alpha_x\cos\left(3\overline{\psi}(s',s)\right)\right)\\ &-\frac{3J_x^{3/2}}{\sqrt{2}\sin(3\pi\nu_x)}\int_s^{s+C} ds' k_x\beta_x^{-1/2}\alpha_x\alpha_x(s)\\ &\times\left(\cos\left(3\overline{\psi}(s',s)\right) - \alpha_x\sin\left(3\overline{\psi}(s',s)\right)\right)\\ &-\frac{J_x^{3/2}}{4\sqrt{2}\sin(3\pi\nu_x)}\int_s^{s+C} ds'\sqrt{\beta_x}(g_1\beta_x - 3\frac{k_x}{\beta_x})\\ &\times\left(2\alpha_x(s)\sin\left(3\overline{\psi}(s',s)\right) - \beta_x(s)\gamma_x(s)\cos\left(3\overline{\psi}(s',s)\right)\right)\end{aligned}$$

where (ϕ_x, J_x) are the action-angle coordinates, $\beta_{x,y}$, $\alpha_{x,y}$ and $\gamma_{x,y}$ the Twiss parameters, $\overline{\psi_x}(s',s) = \phi_x(s') - \phi_x(s) - \pi\nu_x$, and ν_x the tune.

[1] A.Loulergue et al., in Proc. IPAC2018, Vancouver, BC, Canada, 2018.

[2] M.Takao, in Proc. EPAC08, Genoa, Italy.

[3] M.Takao, Phys. Rev. E \textbf{72} (2005), 046502.